34th Annual Mathematics Symposium - Western Kentucky University
October 10 - October 11, 2014

*= student presentation

Friday 4:30pm, Registration and refreshments begin, 1st floor of Snell Hall (SH)

Friday 4:50 pm, ROOM 1108 SH
Welcome by Head of the Mathematics Department: Bruce Kessler

Friday 5:00-6:00 pm, ROOM 1108 (SH)

INVITED TALK

Robert L. Devaney
(Boston University)

The Fractal Geometry of the Mandelbrot Set

In this lecture we describe several folk theorems concerning the Mandelbrot set. While this set is extremely complicated from a geometric point of view, we will show that, as long as you know how to add and how to count, you can understand this geometry completely. We will encounter many famous mathematical objects in the Mandelbrot set, like Farey tree and the Fibonacci sequence. And we will find many soon-to-be-famous objects as well, like the "Devaney" sequence. There might even be a joke or two in the talk.

PARALLEL SESSIONS, Friday

Friday 6:00 - 6:20 pm

ROOM 1101 •• **Daniel King*** (NKU)
Patterns in Pythagorean Triples

Euclid's formula is a fundamental formula for generating Pythagorean triple (PTs) given an arbitrary pair of positive integers, m and n.   The PT is primitive if and only if m and n are relatively prime. Consider the PTs as points in 3-space that lie on the cone $x^2 + y^2 = z^2$.   These points can be thought of as vertices of polygons, particularly right triangles, on the cone. Pursuit of right triangles in 3-space with both PT vertices and integer edges has led to interesting results. This presentation will examine the properties of integer distances between PT points in 3-space.

ROOM 1102 •• **Julia Gensheimer* & Valerie Richmond*** (WKU)
Splines and Physics: An Application to Gaming

Using Mathematica programming we created an interactive roller coaster with an adjustable track, moving cart, coin collection system, levels, and a 2D and 3D viewing mode. We implemented splines for our track and the physics of acceleration due to gravity and friction for the movement of the cart.

ROOM 1103 •• **Nitin Krishna*** (U. Chicago)
Quantifying Limits on Replication, Death, and Quiescence in Mycobacterium tuberculosis in Mice

According to the World Health Organization, an estimated 2.3 billion people are infected with Mycobacterium tuberculosis (Mtb). Despite decades of research, the physiology of tuberculosis remains poorly understood. Current models for Mtb hold that during latter stages of infection, bacteria enter a state of nonreplicating persistence, or static equilibrium. A recent study of Mtb-infected mice by Gill et al. (2009) used a mathematical model to show that bacterial replication and death rates do not necessarily remain constant. In our study, we extend this model by investigating the effects of a time-dependent plasmid segregation rate and the inclusion of quiescence to find limits on growth rates that are consistent with bacteria counts. We find that there are alternative hypotheses to tuberculosis pathogenesis that lead to lower estimates of Mtb replication and death rates. We also show that replication and death rates of Mtb may be higher than initially predicted when bacterial quiescence is added into the model.

Friday 6:30 - 6:50 pm

ROOM 1101 •• **Kain Kotoucek*,** Dana Beichele-Speziale (WKU)
Analyzing the mathematics of mini-golf

We coded a simulation of a mini-golf game in Mathematica, using trigonometric, kinematic, algebraic, and specific algorithmic functions.

ROOM 1102 •• **Stephen Guffey*** (WKU)
Optimal Control of a Parabolic System of PDEs for a Model of Wound Healing

In this talk we discuss a problem arising from the study of chronic wounds. One form of treatment if to use hyperbaric oxygen therapy to supply extra oxygen to the wound. However, if too much oxygen is applied, oxygen toxicity kills the patient's body cells as well as the invasive bacteria. This suggests there is some optimal level of oxygen that can be applied to heal the wound. We take a system of partial differential equations (PDEs) that models the biological interactions inside the wound, and attempt to use methods from Optimal Control theory to find an optimal treatment strategy using supplemental oxygen. Optimal control of PDEs leads us into an investigation of Hilbert Spaces to find our solution.

ROOM 1103 •• **Archie Rowe*** (KWC)
Random Trees: History and Applications

A brief history of random trees and some of their applications: We will discuss rapidly exploring random trees and random binary trees and some of their uses in robotics and computer science online environment. We plan to highlight methods that we feel have created a successful course. The course we will discuss is a conceptual statistics course, but the methods used could be applied to a variety of courses in different disciplines.

Friday 7:00 - 7:20 pm

ROOM 1101 •• **John E. Mosley*** (UK)
Greatest Common Divisor of Multinomial

While studying a particular characteristic number of SU-manifolds we came across a fun and interesting number theory problem. In this presentation, I will give the answer to this problem, which gives the greatest common divisor of certain multinomial coefficients.

ROOM 1102 •• **Rebecca Crouch* & Amanda Ellis*** (UK)
Experiences in Transitioning a Blended Classroom to an Online Environment

In recent years, educators have seen a higher demand for online education. Online education offers students and educational experience that allows then to work in various locations and at various times. This demand for online education has sparked the need for classes that were traditionally offered in an in-class setting to be transitioned into an online environment. This transition to the online environment can be difficult for instructors, especially in classes that were initially developed to be taught in an inverted setting. In this talk, we plan to discuss our experience in transitioning a classroom originally taught in a blended model to an online environment. We plan to highlight methods that we feel have created a successful course. The course we will discuss is a conceptual statistics course, but the methods used could be applied to a variety of courses in different disciplines.

ROOM 1103 • **Sam Booth*, Meredith Bickett*** (WKU)
Flypes and their Relation to Loop Number in Knots

During this talk we will give a brief introduction to knot theory while specifically focusing on loop numbers. Next we introduce 4-plats and flypes which change one knot diagram to another diagram of the same knot. We attempt to show a relationship between flyped knot diagrams of 4-plats and the maximum, average, and minimum loop numbers in their diagrams.

Friday 7:30-7:55 pm Food and Refreshments!

Friday 8:00 - 8:20 pm

ROOM 1108 •• **Actors: Zachary Bettersworth*, Samantha McKean*, Zachary Pennigton*, Alex Malon*, Dimitri Leggas*, Mahannah El-Farrah*, Jeremy Bivins*, Meredith Bickett*,** Claus Ernst (WKU)
Original plays by: C. Adams, freely adapted by M. El-Farrah, J. Bivins and C. Ernst.

Mathematical Comedy (in two acts)

Act 1. A difficult delivery
Act 2. Group Therapy

Friday 8:25 - 8:55 pm

ROOM 1108 •• **Panelists: Emily Greenwood, Whitney Miller, Rob Sparksman, Christopher McMahan, Rebecca Crouch, Chase Russell**

Panel session: Careers in Mathematics

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PARALLEL SESSIONS, Saturday

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Saturday 8:30 - 8:50 am

ROOM 1101 •• **Anthony Della Pella*** (U. MI-Dearborn)
Zeroes of weighted Bergman kernels

Let D denote the unit disk in C, and $B_\lambda(z,w)$ denote the weighted Bergman kernel corresponding to a weight λ on D. It is a nontrivial task to determine whether or not equipping a weight λ to a Bergman space causes the kernel function to vanish, as many of the well known weights do not give rise to zeroes. In this project, we exhibit certain weights for which the kernel function $B_\lambda(z,w)$ indeed vanishes.

Identifying properties of the weighted Bergman kernel for varying weights is currently an active research field, and in particular, whether or not these kernel functions contain zeroes is of interest to us. Containing zeroes is an aspect of the kernel function in one dimension which readily generalizes to higher dimensions where this question relates to certain geometric invariants.

In addition to precise theoretical results, we obtain certain numerical results. Through the creation of a program which utilizes the Sage mathematical software and the Python programming language we further analyze weighted Bergman kernels corresponding to more general weights.

ROOM 1102 •• **Kristen Knight*** (Purdue)
Deterministic and stochastic models of methicillian-resistant staphylococcus aureus transmission in the hospital setting

Both deterministic and stochastic models are derived to quantify the interactions of community and hospital-acquired methicillin-resistant staphylococcus aureus transmission in the hospital setting. The reproduction number is determined and sensitivity analysis is carried out to determine the impact of model parameters on the spread of the bacteria. The transition rates, Kolmogorov-forward equations, and cumulants of the continuous-time Markov chain are derived. Numerical simulations of the models show that disease prevalence among patients and healthcare workers are impacted by hand washing compliance rate, the healthcare workers decolonization rate, environmental contamination rate, the admission rates into the hospital and isolation rate of colonized patients.

ROOM 1103 •• **Christopher S. McMahan** (Clemson)
The Bayesian CAR model: Modeling the incidence of Lyme disease within the conterminous United States

Lyme disease, at tick borne infection, is caused by the bacterium Borrelia burgdorferi. Within the United States (U.S.), the transmission of this disease, to both humans and canines, is commonly attributed to two ixodid ticks; specifically, the primary vector in the upper Midwest and Northeast is the black-legged tick (Ixodes scapularis), whereas the western black-legged tick is the primary vector in the West. In this presentation we will explore key factors that are believed to be related to the prevalence of this disease within both human and canine populations. Our work explores spatial, environmental, economic, and vector-born factors that have been suggested to influence Lyme disease prevalence rates. To account for spatial dependence, the conditional autoregressive (CAR) model was chosen as a prior distribution for spatially structured random effects in a multivariate generalized linear model. This model is then fit to county-by-county data consisting of the number of cases of both human and canine Lyme disease. The results from this analysis allow us to assess factor importance, construct disease maps that indicate areas of high risk, and quantify the county specific correlation that exists between the number of human and canine cases of this diseases.

Saturday 9:00 - 9:20 am

ROOM 1101 •• **Hannah Pennington*** (WKU)
A mathematical model for the interactions of MMP-1, TIMP-1, and ECM in a healing wound

In this work, we aim to quantify parameters for a mathematical model describing interactions among proteins in a diabetic wound, including matrix metalloproteinases (MMPs) and their inhibitors (TIMPs); the extracellular matrix (ECM); and fibroblasts in a diabetic wound. By comparing the data taken from Muller et al., we create a functional which minimizes the sum of the squares of the error. Using MATLAB's "globalsearch" and "fmincon" functions, we minimize the functional by selecting an initial set of parameter values and having MATLAB identify a local minimum value. Sensitivity analyses are conducted to measure to what degree the equations were affected by slight changes in the model. A sensitivity analysis utilizing Latin hypercube sampling and Partial Rank Correlation Coefficient procedure (LHS/PRCC) can be used in combination to ascertain uncertainty of the model over a global parameter space. The results of the analyses are used to assess the biological significance of the parameters in relation to each compartment of the model to further understand its biological implications.

ROOM 1102 •• **Mark Robinson** (WKU)
"Analysis of Some Systems of Differential Equations"

The behavior of solutions to nonlinear systems of differential equations of the form $dx/dt = f(x,y)$, $dy/dt = g(x,y)$ is considered. The analysis of a linear system and the classification of its equilibrium points are discussed. Analysis of a nonlinear system is accomplished by linearizing the system near each equilibrium point. Examples will be provided, including a nonlinear predator-prey model.

ROOM 1103 •• **Aquia Richburg*** (Morehouse College)
Forbidden Subgraphs of Competition Graphs

Since the introduction of competition graphs much work has been done to study their properties. In this paper we look at classifying forbidden subgraphs of competition graphs of doubly partial order. These results also extend to n-tuply partial order. This paper also looks at the correlation of Dyck paths, Catalan numbers and partially ordered sets.

ROOM 1101 •• **Lindsey Walton* & Saralinda Schell** (WKU)
Lord of the Points

Given several 3-dimensional objects as 3D datasets (that is as a cloud of points from a 3D-scanner), we show how to align the datasets using a technique called "Generalized Procrustes' Analysis". We show how to select subsets of data using a convex hull operation in Mathematica and R.

ROOM 1102 •• **Adam Volk*** (U. Dayton)
Gridline graphs in higher dimensions

A gridline graph is a graph G whose vertices can be realized in R^2 in such a way that the vertices are adjacent if and only if they lie on a vertical or horizontal line, in other words they share a coordinate. These graphs can be characterized as line graphs of bipartite graphs and as diamond, claw, and odd-hole free graphs. We generalize gridline graphs to n dimensions by letting vertices be adjacent whenever they line on the same hyperplane, that is they share at least one coordinate. We investigate how characterizations of the 2-dimensional gridline graphs generalize to n dimensions and provide more specific results for the case where n = 3. This research was conducted as part of the 2014 REU program at Grand Valley State University.

ROOM 1103 •• **Sanju Gupta** (WKU)
A topological twist on materials science: *De novo* carbons as posterchild and beyond

The objective of this presentation is twofold: 1) to address the key concept of topology that impacts materials science in a major way and 2) to convey an excitement of the recent significant advances in our understanding of the important topological notions in a wide class of biological and multifunctional materials with potential technologies. A paradigm of topology/geometry -> property -> functionality is emerging that goes beyond the traditional paradigm of microscopic structure -> property -> functionality relationship. The new approach delineates the active roles of topology and geometry in design, fabrication, characterization, and predictive modeling of novel materials properties and multifunctionalities. After introducing the essentials of topology and geometry, we elucidate these concepts through a gamut of de novo carbon allotropes, hierarchical self-assembled soft- and biomaterials, supramolecular assemblies, nanoporous materials, and so forth. Applications of these topological materials range from sensing, energy storage/conversion, and catalysis to nanomedicine. We illustrate these concepts through resonance Raman spectroscopy on nanocarbons as well as a slew of soft and biomaterials with an emphasis on topological metrology. We also discuss topological defects (local versus extended) such as Stone-Wales and mitosis, domain walls, vortices, boojums, skyrmions, magnetic monopoles in artificial spin ice, and Hopf fibrations, as well as other (complex and network) topologies in advanced materials and quantum systems.

ROOM 1101 •• **Claus Ernst** (WKU)
Strategies to unknot a tangled string

There are two different strategies one could use to unknot: Strand-passage and nullification. We show that both of these are employed by nature in the world of tangled DNA.

ROOM 1102 •• **Andrew Wilson***, Logan Yliniemi (Austin Peay State University)
Evolutionary multi-objective mutiagent credit assignment

Multiagent multi-objective systems are important to optimize because of their significance in many real world problems. However, prior to this research no work existed pertaining to credit assignment in multi-objective systems. We introduced difference rewards as a form of credit assignment to NSGA-II and SPEA2, two well known evolutionary algorithms in multi-objective optimization. We found that the multiagent multi-objective system was far more optimized when using difference rewards than using simply the global reward. Proper credit assignment is crucial for optimizing these systems.

ROOM 1103 •• **Dimitri Leggas***, **Andrew Brown*** (WKU)
Finding balanced and separable partitions of three sets

Let A, B and C be three disjoint sets of points on the real line such that $|A|=|B|=|C|$. Partition A, B, and C such that $A = A_1 U…UA_k$, $B = B_1 U…U B_k$ and $C = C_1 U…UC_k$. This is called a balanced partition if $|A_i| = |B_i| = |C_i|$ for every $1 <= i <= k$. The partition is separable if the convex hulls of A_i, B_i, and C_i are pairwise disjoint for every $1 <= i <= k$. What is the minimal k such that there exists a balanced and separable partition of the sets into k parts? We know that if k = 5 then such a partition always exists. For k = 3 we know an example of three sets of equal cardinality such that this partition does not exist. Therefore the lowest possible value of k for which a balanced and separable partition exists is either 4 or 5. We used a new approach to find such a partition for k = 4 by showing that certain linear programming problems are not feasible. We used an algorithm that works with an order of separators to produce a matrix of inequalities that must be satisfied for the partition to be possible with k = 4. If there is no solution to this system, then the algorithm works for that order of separators. We produce all of the valid orders of separators and check that the algorithm works for each of them, exhaustively showing that the minimum k is k = 4 for all choices of A, B, and C.

Saturday 10:30 - 10:50 am

ROOM 1101 •• **Katrina Ferrier*** (UTC)
The Complex Zeros of a Gaussian Random Polynomial

Let $P_n(z)$, where z is a complex variable, be a polynomial whose coefficients are independent, identically distributed and normalized real Gaussian random variables. Let Ω be any Lebesgue measurable subset of the reals and denote by $v_n(\Omega)$ the number of zeros in Ω of $P_n(z)$. In 1943, Kac obtained an explicit intensity function $g_n(x)$ for which the expectation of $v_n(\Omega)$ is given explicitly by $\int_\Omega g_n(x)dx$ for each n>1. In 1995, Shepp and Vanderbei extended Kac's result to the case when Ω is any Lebesgue measurable subset of the complex plane. In this talk, I will consider the case when the coefficients are independent, identically distributed and normalized complex Gaussian random variables and use the method of Shepp and Vanderbei to obtain an explicit intensity function $h_n(z)$ expressed in the simplest term for which the expectation of $v_n(\Omega)$ is given explicitly by $\int_\Omega h_n(x)dx$ for each n>1. I will also present numerical computation that demonstrates the behavior of $h_n(z)$ for various values of n and limiting expressions for $h_n(z)$ and the expected number of zeros in disks and sectors of the complex plane. (Based on joint work with Micah Jack.)

ROOM 1102 •• **Uta Ziegler** (WKU)
Algebraic knots and trees

This talk demonstrates a coloring method for Conway Algebraic knot diagrams and shows how a colored knot diagram is translated into a binary tree. Once the binary tree is created we can apply approaches developed for trees, e.g. tree-traversal to save the diagram information or embedding of the tree in a 2D grid. Knots and Trees.

ROOM 1103 • **Thomas Richmond** (WKU)
Instant Insanity II

Instant Insanity is a puzzle requiring the alignment of four 1 x 1 cubes with colored faces in a row so that none of the resulting 1 x 4 faces has a repeated color. It was popular in the 1970s and its solution appeared in graph theory texts. The Instant Insanity II puzzle requires the alignment of 4 row of colored tiles so that each row has no repeated colors, but the tiles slide along tracks on a cylinder. The manufacturer claims that there are only 24 correct solutions. We will use simple graph theoretic techniques to show that there are 48 solutions.

Saturday 11:00-11:30 am, Refreshments

Saturday 11:30 am- 12:30 pm, ROOM 1108 (SH)

INVITED TALK

Louis H. Kauffman
(**University of Illinois at Chicago**)

Knots and Physics

We will talk about the remarkable interweaving of themes and results between the theory of knots and physical theory. Knot theory begins as a mathematics that models the topological behaviour of rope, and so is at its inception related to the physics of rope. If we go beyond topology to add forces and friction for ropes, we find ourselves in relatively uncharted research territory almost immediately! So our talk will begin by demonstrating problems in this area. There are other directions. Lord Kelvin in the 1800's theorized that knotted vortices in the luminiferous aether were the source of atoms in the material world. This hypothesis does not die, but it has changed its form since Kelvin's day. The luminiferous aether has been replaced by space-time and by fields. Physicists speculate that knotted fields (gluon fields forming quantum knots) may actually occur. Knotted vortices in familiar fluids such as water, have been produced in the laboratory by William Irvine and his group at the University of Chicago. The topology of knots and links has many interconnections with quantum theory and statistical mechanics. We will talk about these connections with the help of the Kauffman bracket model for the Jones polynomial. These directions reach toward new invariants such as Khovanov homology and subtle relationships of knot theory with string theory. Finally, we cannot resist the fascinating applications of knot theory to Magic. In this domain the topology interacts with slights of hand, and anything can happen.