35th Annual Mathematics Symposium - Western Kentucky University
November 13 – November 14, 2015

*= student presentation

Friday 3:30pm, Registration and refreshments begin, 1st floor of Snell Hall (SH)

Friday 3:50 pm, ROOM 1108 SH
Welcome by Head of the Mathematics Department: Bruce Kessler

Friday 4:00-5:00 pm, ROOM 1108 (SH)

INVITED TALK

Henry Segerman
(Oklahoma State University)

Design of 3D printed mathematical art

When visualising topological objects via 3D printing, we need a three-dimensional geometric representation of the object. There are approximately three broad strategies for doing this: "Manual" - using whatever design software is available to build the object by hand; "Parametric/Implicit" - generating the desired geometry using a parametrisation or implicit description of the object; and "Iterative" - numerically solving an optimisation problem.

The manual strategy is unlikely to produce good results unless the subject is very simple. In general, if there is a reasonably canonical geometric structure on the topological object, then we hope to be able to produce a parametrisation of it. However, in many cases this seems to be impossible and some form of iterative method is the best we can do. Within the parametric setting, there are still better and worse ways to proceed. For example, a geometric representation should demonstrate as many of the symmetries of the object as possible. There are similar issues in making three-dimensional representations of higher dimensional objects. I will discuss these matters with many examples, including visualisation of four-dimensional polytopes (using orthogonal versus stereographic projection) and Seifert surfaces (comparing my work with Saul Schleimer with Jack van Wijk's iterative techniques).

I will also describe some computational problems that have come up in my 3D printed work, including the design of 3D printed mobiles (joint work with Marco Mahler), "Triple gear" and a visualisation of the Klein Quartic (joint work with Saul Schleimer), and hinged surfaces with negative curvature (joint work with Geoffrey Irving).

Friday 5:00-5:20 pm Refreshments!

PARALLEL SESSIONS, Friday

Friday 5:30 - 5:50 pm

ROOM 1101 •• **Rodrigo D. Daboin Sanchez*** (WKU)
TwixT—A Strategy Game and Artificial Intelligence

TwixT is a two player game in which one player builds a fence from left to right and the other from top to bottom. The winner is the player who completes his or her fence. Obviously, once one fence is built the other fence can't be built since fences are not allowed to cross. In the presentation, I will address how I (and my partner at the time) approached making a computer program for the game, including its practical and theoretical aspects. The conception of an artificial intelligence for TwixT will also be discussed, particularly the strategies adopted and their implementation. Concluding remarks will be centered on how this AI can be improved and the future implementation of these new ideas.

ROOM 1102 •• **Johnathan Webb* & Mitchell Webster*** (WKU)
Evaluating the Thermodynamic Models of Heat Transfer

In this presentation we will explore the thermodynamics of the home brewing process. Home brewing beer generally follows the same process of commercial brewing just on a much smaller scale. Both process involves mashing the milled grains with hot water in a lauter tun at a specific temperature. The temperature of the mash is maintained as close as possible for the duration of the mashing process, typically 60-90 min. A typical home brew lauter tun consists of an insulated container that minimizes heat loss, without direct temperature control. A modified cooler is a popular choice. Using differential equations we will be analyzing the theoretical heat loss of our modified cooler lauter tun. Once we have our theoretical values for heat loss we will test this value by measuring the temperature of the mash throughout the mashing process. We will then find the optimal dimensions for our lauter tun to minimize the heat loss during the mashing process.

ROOM 1103 •• **Gulnisa Karadshayeva*** (WKU)
Expectations When Harvesting Cherries

In this talk, we will give a short introduction to the children's game *Hi Ho! Cherry-O* and how Markov chains can be used to model this game of chance. Using Markov chain properties, we construct an 11×11 transition matrix, **P**, for *Hi Ho! Cherry-O* and use it to answer the following questions: What is the expected length of a game for a single player? What is the expected length of a game when multiple players are considered? What happens to the expected length of a game if we modify the rules and/or setup of the game?

Friday 6:00 - 6:20 pm

ROOM 1101 •• **Elizabeth Pulsifer*, Esther Huggins*** (WKU)
Development and Implementation of a Gel Electrophoresis Simulator Using Mathematica

Gel electrophoresis is a standard tool in molecular biology. The presenters developed a simulator of this process as both a learning tool and a virtual gel image generator for Western Kentucky University's Genome Discovery and Exploration Project and Computational Problem Solving class. This Mathematica®-coded program was designed to realistically mimic the mechanics of agarose gel DNA electrophoresis with specifications tuned to mycobacteriophages. The program has multiple aspects to either experiment with an already sequenced mycobacteriophage with an optimal comparison, provide two files that contain the genome sequence for comparison, or generate a virtual gel from a sequence file. As an empirical analysis and proof of accurate displays, the program was tested to replicate comparable sequenced mycobacteriophages gel figures from a database. This talk will explain how the program works and how it can be used.

ROOM 1102 •• **Dan King** (NKU)
Modeling Ebola Virus Disease

Originally a problem from the 2015 Mathematical Contest in Modeling, this talk describes the process of creating a realistic model of the recent Ebola Virus Disease outbreak in West Africa. Beginning with a system of differential equations known as an SIR model to represent the disease, additional equations and parameters are added to the system to describe the various stages of the disease with more precision. Once a realistic model has been set, the focus shifts to possible actions that can minimize the spread of the disease such as implementation of a hypothetical vaccine or quarantining.

ROOM 1103 •• **Courtney P. Inabnitt*** (WKU)
Viviani's Theorem in Higher Dimensions

Viviani's Theorem states that if *P* is a point on the interior or one of the sides of an equilateral triangle then the sum of the distances from point *P* to the sides of the equilateral triangle equals the length of its altitude. This project extends on Viviani's Theorem by exploring what happens when a point is on the exterior of a regular polygon. The project aims to find a generalization for a point on the exterior of a regular polygon in relation to the number of sides and the apothem. In addition, this project will explore Viviani's Theorem further by considering the relationships between the distances from a point on the interior of a regular polyhedron to its respective faces and the height of the polyhedron.

Friday 6:30 - 6:50 pm

ROOM 1101 •• **Michael Belcher*** (WKU)
Modeling Tumor Growth Using Discrete Fractional Calculus

The main purpose of this research project is to improve the Gompertz curve to create a more accurate and precise model of tumor growth that can be personalized for each patient. To do this we first calculate the growth rates for mice data which has been provided us by a health care professional in Nashville. The shape of the growth rates will direct us to choose the model which will fit better with the data. We will also use a recent theory which is called discrete fractional calculus to improve the existence models in the literature. The effectiveness of the models will be checked by using some statistical techniques such as sum of residual squares. We will intensively use a software "Mathematica" in our research.

ROOM 1102 •• **Cameron White** (WKU)
Enough is Enough: The Mathematics Behind Knowing When to Quit

In what came to be known as 'The Secretary Problem,' the question posed by Martin Gardner was a relatively simple one: given a random set of numbers, along with a few decision rules, what is the best strategy to maximize the number chosen?

In this talk I will discuss how and why the Secretary Problem became the Secretary Problem, as well as a few of the different, interestingly named variants in the more general field of optimal stopping. The basic rules of the Secretary Problem are introduced, and then, using the statistical software language R, we can define a function that randomly simulates the hiring of a secretary using the aforementioned stopping rule. In the case of one secretary, we can, using nothing more than elementary calculus and basic probability, find a closed form function, $f(x)$, that gives the probability of hiring the 'best' secretary for each stopping point x.

But what if we wanted to hire two secretaries? Three secretaries? k secretaries? I will cover those situations as well, at first by simulation, and then later by conjecturing a closed form function $f(k,x)$ that will return the probability of hiring the top k secretaries given a stopping point x. The addition of a potential cost will be included in our simulation, and then, lastly, some discussion on possible future research opportunities in optimal stopping will be discussed.

ROOM 1103 •• **Kathleen Bell*** (WKU)
Transitions of Topological Relations between Planar Regions

As two topological regions are morphed and translated, how does their intersection change? I study what forms of intersection are possible, and what transitions are directly possible, while considering such variables as the connectedness of the regions.

ROOM 1108 •• **Mark Reid** (JHUAPL)
The Automation of NASA's New Horizons Mission to Pluto

This presentation will discuss the application of mathematics to autonomous space exploration, with particular focus on the onboard Autonomous Operations and Fault Management of the New Horizons mission. Topics covered will focus on applied discrete mathematics and Boolean logic.

Friday 7:00 - 7:30 pm

ROOM 1101 •• **Luke Miles*, Niven Achenjang*** (WKU)
A Universe Inside Mathematica

A physics engine is a program that takes an arbitrary starting state and runs it under some laws for as long as needed. We have used Mathematica to make a simple space physics engine focusing on collisions and efficient gravity calculation. Collisions can be between solid or gaseous bodies, and gravity is made faster using quadtrees and hashes.

ROOM 1102 •• **Andrew Davis*** (WKU)
A Logistic Version of the Leslie Matrix for Population Modeling

Leslie matrices are used to model populations, and have the advantage of giving the population size of disjoint age groups within the population, based on birth rates and survival rates for each age group. However, this is a discrete exponential model, with time step equal to the age group interval length, and does not account for carrying capacity like logistic models. Attempts have been made to create a logistic version of the Leslie matrix model, but they have had limited success. We have developed a Leslie-like matrix model that exhibits discrete logistic behavior with regard to population size, with the advantage of predicting the population size among age groups. The model uses a Leslie matrix, along with a population-dependent adjustment matrix that contains factors to stabilize the total population size to the carrying capacity and the population distribution to the desired distribution vector. This model is very similar to the discrete logistic model, in that the dominant eigenvalue of the Leslie matrix acts much like the growth rate factor in the discrete logistic model, and exhibits very similar behavior in population modeling. We believe that this model has many applications in ecological studies.

ROOM 1103 •• **Hannah Keith*** (WKU)
The Probabilities of Building an Empire

In this project, the random probabilities associated with the modernized (and much different) Monopoly Empire game are explored through the medium of Markov chains. In addition to accounting for a non-standard die, matrices consider the effects of Chance cards and the Go to Jail space. By calculating limiting probabilities for each space on the board, the researcher was able to determine which companies are most likely to be landed on throughout the course of the game. Ultimately, these probabilities help to answer the question: "Which companies on the board produce the greatest minimum expected return?"

ROOM 1108 •• **Richard Schugart** (WKU)
Connecting Local and Global Sensitivities for a Mathematical Model in Wound Healing

A mathematical model for proteolytic enzyme interactions and their effects on the healing response of a wound will be presented. The differential-equation model was curve fit to averaged data of patients with diabetic foot ulcers. Multiple sensitivity analyses were conducted. Results from a Latin hypercube sampling combined with a partial correlation coefficient analysis will be shown and compared to a local classical sensitivity analysis leading to a local-to-global analysis of our parameter space.

Friday 7:30-7:55pm Food and Refreshments!

Friday 7:55 - 8:25 pm

ROOM 1108 •• **Panel Session: Careers in Mathematics**
Panelists: Cameron White (Fruit of the Loom), Mark Reid (JHUPL), Samantha McKean (Vanderbilt), Lindsey Brown (Simpson, KY schools)

Friday 8:30 - 9:00 pm

ROOM 1108 •• **Mathematical Play: "Math Talk"**
Actors: Brothers TI and Proof and various callers
Authors: Alex Malone, Zach Pennigton, Zach Betterworth, Mahannah El-Farrah, Luke Miles, Molly Dunkum and Claus Ernst

Abstract: Mathematical comedy in one act. This play will follow the style of the popular show "Car-talk" on NPR. But instead of car questions our show will deal with math questions.

Saturday from 8:00 am - Registration and refreshments, Third Floor of Snell Hall (SH)
REGISTRATION continues until 11:30am SATURDAY

PARALLEL SESSIONS, Saturday

Saturday 8:30 - 8:50 am

ROOM 3106 •• **Mahannah El-Farrah** (WKU)
Expectation Numbers of Cyclic Groups and Values of the Riemann zeta function

The average order of the elements of a cyclic group has been well-studied in recent years. We consider the average order of the subgroups of a cyclic group which are generated by a fixed number of elements. Values of the Riemann zeta function arise in expressing the average size of such subgroups.

ROOM 3107 •• **Peter Agaba*** (WKU)
Incorporating a Discount Factor in an Optimal Control Model of Chronic Wound Healing

A chronic wound is a wound that does not heal in an order and on time. The analysis in this research project focuses on treating chronic wounds using both mathematical and biological models. These models primarily focus on the amount of oxygen supplied to the wound. This amount should be optimal since too much oxygen is toxic to the body, and can potentially lead to death. Some other models focus on capturing the significance of the length of therapy in wound treatment. Our goal is to minimize the time used in therapies since longer periods make treatments more costly. In order to do this, we are incorporating discount factors into several models of wound healing.

ROOM 3110 •• **Uta Zeigler** (WKU)
Random polygon model to study packed viral DNA

Knotted entities occur in polymer chains in various life sciences such as physics, chemistry and biology. The example of a polymer chain most people are familiar with is DNA – though the 'knotted' part comes as a bit of a surprise. The (temporary) knot configurations captured in DNA are the results of manipulations of this DNA. Information about knot configurations captured in the DNA allow conclusions to be made about the potential manipulations. This presentation focuses on a mathematical investigation which was motivated by biologists studying the packing of viral DNA inside the virus 'heads' during the assembly process of the viruses. From experiments it is known that DNA forcible removed from mutant P4 viruses is often highly knotted. In the presented research, viral DNA is modeled as freely-joined, unit-length polygons, and the virus 'head' is a sphere. The presentation explains the underlying model and some results.

Saturday 9:00 - 9:20 am

ROOM 3106 •• **Mark Robinson** (WKU)
Topics in the Numerical Solution of Differential Equations

Several difference methods that are used for the numerical solution of initial-value problems in ordinary differential equations are examined. Topics of interest that are investigated include the connection between certain difference schemes and numerical methods for approximating integrals, difficulties that may be encountered by difference schemes, and appropriate selection of methods.

ROOM 3107 •• **Lauren Otterbach*** (WKU)
Fractal Interpolation Functions and Their Integrals

This presentation discusses fractal interpolation functions. Although fractal interpolation functions are not necessarily differentiable, they are continuous causing them to be integrable. This presentation addresses how to arrive at the integral of a fractal interpolation function.

ROOM 3110 •• **Eura Shin*, Jenna Ellis*** (WKU)
The Development of a Computer Program to Simplify Complex Knot Diagrams using Global Moves

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A mathematical knot is similar in concept to the everyday head phone cable, with the ends closed together to form a continuous loop. These knots are the subject of discussion in molecular biology, mathematics, physics, and chemistry. This presentation will include a brief introduction to knot theory, focus on the problem of knot simplification, cover algorithms developed to represent simplification moves numerically, and discuss the approach used to determine the order in which each move is performed to lead to an optimal simplification. In addition, this program brings what we have coined "global moves" to the table, a largescale approach to minimizing knot diagrams more effectively. The impact the global move has on the simplification process is tested in the form of a "global slide" and the results are compared to that of a previous simplification program.

Saturday 9:30 - 9:50 am

ROOM 3106 •• **Aykut Arslan*** (WKU)
Ribbon Surgery

In this presentation we show that zero surgery detects the existance of the rib- bon structure of a knot. In other words if zero surgery along a ribbon knot K and zero surgery along another knot L gives homeomorphic 3-manifolds then L has to be a ribbon knot too. Moreover, we prove that these ribbon knots have diffeomorphic ribbon disk exteriors $D^4 - D_K^2 \sim= D^4 - D_L^2$ for some ribbon disks D_K^2 and D_L^2.

ROOM 3107 •• **Richard Schugart** (WKU)
Using Optimal Control Theory to Analyze the Treatment of a Bacterial Infection in a Wound with Oxygen Therapy

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A mathematical model describing the interactions of bacteria, inflammatory cells, and oxygen in a wound describing the treatment of a bacterial infection using oxygen therapy will be presented and analyzed. A second variation of the model will be presented in an optimal control setting with the control variable being the input of supplemental oxygen. Numerical results of the optimal control model will be presented and future directions will be discussed.

ROOM 3110 •• **Donald Price*** (WKU)
Nullification of Knots and Ways of Minimizing the Nullification Number

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This study considers two different nullification numbers that are based on diagrams of knots. Nullification moves are local changes made to a crossing in a knot diagram that change the knot diagram to a trivial knot diagram. The number of such moves needed is called the nullification number of the knot diagram. We use the PD Code to represent knot diagrams and our algorithms make nullification moves using PD codes. We looked at which form of nullification tends to be better or worse in terms of how many nullifications moves are needed to end up with the unknot.

Saturday 10:00 - 10:20 am

ROOM 3106 •• **David Roach** (Murray State)
What is a wavelet? A brief introduction

In this accessible talk for a general audience, the basic properties of wavelets will be presented. Additionally, the application of wavelets to image compression will be demonstrated.

ROOM 3107 •• **Attila Por** (WKU)
Almost convex curves

A curve γ in R^d is a continuous mapping of a closed interval. We call γ (≤ k)-crossing if it intersects every hyperplane at most k times. There are no (≤ (d − 1))-crossing curves in R^d since any d points are on a hyperplane. The (≤ d)-crossing curves are often called convex curves like the moment curve $\{(t,t^2,\dots,t^d) \mid t \in [0,1]\}$.

Theorem 1. For all d ≥ 2 there exists M(d) such that every (≤ d+1)- crossing curve in R^d can be subdivided into at most M(d) convex curves.

This result implies a tight lower bound for order type homogenous subsequences of points based on previous work of Elias, Roldan, Safernova and Matousek. For a sequence $P = (p_1, \dots, p_n)$ we say it is order type homogeneous if every (d + 1)-tuple has the same sign (orientation). A sequence $P = (p_1, \dots, p_n)$ in R^d is strong order type homogeneous if for every 1 ≤ k ≤ d the projection of P onto the k-dimensional space spanned by the first k coordinates is order type homogeneous. By Ramsey theory there exists a least integer $N = OT_d(n)$ such that every sequence of points $P = (p_1, \dots, p_N)$ in general position has an order type homgeneous subsequence of length at least n.

Theorem 2. (Suk) $OT_d(n) \le twr_d(O(n))$ where $twr_d(a)$ is the tower function of hight d.

Similarly by Ramsey theory there exists a least integer $N = OT^*_d(n)$ such that every sequence of points $P = (p_1, \dots, p_N)$ in general position has a strong order type homgeneous subsequence of length at least n.

Theorem 3. (Elias, Matousek, Rolando, Safernova) $OT^*_d(n) \ge twr_d(n - d)$

Our result implies that Theorem 4. $OT_d(n) \ge OT^*_d(\Omega(n))$ and establishes a tight lower bound for $OT_d(n)$.

This is joint work with I. Barany and J Matousek.

ROOM 3110 •• **Zachary Bettersworth*** (WKU)
Incoherent Nullification of Torus Knots

There are two different strategies one could use to unknot: Strand-passage and nullification. We show that both of these are employed by nature in the world of tangled DNA.

Saturday 10:30 - 10:50 am

ROOM 3106 •• **Menevse Eryuzlu*** (WKU)
Filippov and C-Fillipov

In a typical differential equation the rate of change or velocity is specified at each value of the time t by some function f. In many applications in subjects such as biology and physics, it may be impossible to specify this rate of change in such a deterministic matter. We use Filippov's operation to deal with this problem. Besides, we generalize the Filippov definition and define the C-Filippov. Then we look for the differences between the Filippov and the C-Filippov of a function.

ROOM 3107 •• **Dominic Lanphier** (WKU)
Combinatorial identities from analysis

Combinatorial identities can be shown in a variety of ways. Typically, one uses counting methods, but one can also use complex analysis. As an illustration, we give an integral formula with applications to combinatorial identities, orthogonal polynomials, and special functions. The proof relies on a generalization of Cauchy's beta integral.

ROOM 3110 •• **Claus Ernst** (WKU)
Loop numbers in knot diagrams

For a given knot diagram D one can traverse the knot diagram and count the number of loops created by the traversal. The number of loops created depends on the starting point in the diagram D and on the traversal direction. Looking at the minimum or maximum number of loops over all starting points and directions one can define loop numbers of the diagram D. If one looks over all minimal diagrams D of a knot type these loop numbers become knot invariants. In this talk we make some elementary observations about such loop numbers.

Saturday 11:00 am- 12:00 pm, ROOM 3110 (SH)

INVITED TALK

Mark Reid
(**Johns Hopkins University Applied Physics Laboratory**)

The Mathematics of Space Exploration – A Look at NASA's New Horizons Mission to Pluto

This presentation will provide an overview of the New Horizons Mission to Pluto and discuss the application of various mathematics concepts to autonomous space exploration, with particular focus on the onboard Autonomous Operations and Fault Management of the New Horizons mission. Topics covered will include everything from basic algebra, trigonometry, linear algebra, applied discrete mathematics and Boolean logic.

Saturday 12:30 - 1:30 pm, ROOM 3110 (SH)
After Event by the Math Club: Origami

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