Randall Harper Research Award Application

Lars Hebenstiel

For the past 2 years I have been working with Dr. Novikov to study stochastic resonance in the Duffing oscillator. Stochastic resonance (SR) is a form of resonance in bistable systems where by adding a random external noise force one can induce resonance with a periodic force on the system. Examples of systems where this effect can be found are in climate science, mechanical and electrical engineering, optics, and various chemical and sociological models.

To study this effect, we first had to create a numerical model of the oscillator. To do this, I coded a Jupyter notebook to numerically solve the Duffing differential equation. With this data, it was possible to analyze the behavior of the oscillator through phase portraits (Fig 1), and through Poincaré maps (Fig 2). To study SR however, takes a lot more computational power. Every data point of oscillator response vs noise in the plot of SR behavior (Fig 3) takes a new simulation to calculate, and since the simulation is dependent on stochastic forces, the error in the calculation decreases as $\frac{1}{\sqrt{T}}$ where T is the simulation time. To get around this, I refactored the simulation code to run 64 separate noise strengths in parallel using MPI on WKU's High Performance Computing Center. Even with this many parallel calculation threads, a whole plot with a reasonable error due to noise still takes multiple hours to fully calculate and storing the raw oscillator position vs time data generated by this computation takes up upwards of 80 GB of storage space. These findings were presented at various conferences during 2019-2020.

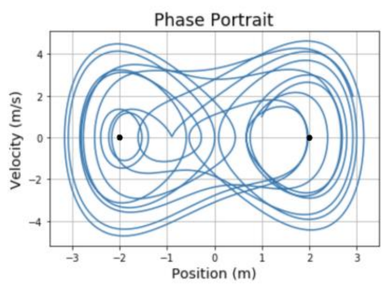

Figure 1. Phase portrait of DDDO with wells marked as black points

Figure 2. Poincaré map of DDDO

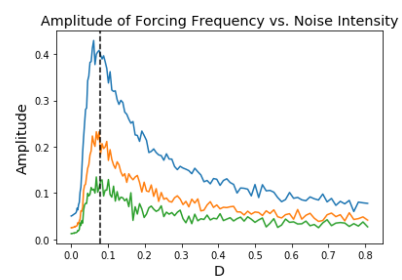

Figure 3. Stochastic resonance behavior of the Duffing oscillator at varying forcing strengths (blue > orange > green)

With this numerical model, Dr. Novikov and I applied for the Kentucky Academy of Science Undergraduate Summer Research Grant and received a $3,000 grant to develop a mechanical model of the Duffing oscillator to study SR in. To do this, I designed a mechanical model in CAD software based on an experimental design used by *Donoso, Ladera, Eur. J. Phys. 33 (2012)*. To improve on their design, our design is reinforced with 8020 aluminum shafts, and we acquire position data with 2 type of position sensors: a force sensor. With guidance from Dr. Harper, I am creating LabVIEW code to control the model and acquire data synchronously with the control of the external forces applied in the model.

While stochastic resonance has repeatedly been shown to exist in some experimental systems, overall data in simpler systems and comparison to numerical models is lacking. With this experimental data, we will match the data produced by the experimental model, to the numerical model. We are currently working on a manuscript describing our developed system and results and are working towards making the setup available for undergraduate labs in physics.