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Information concentration along the boundary contours of naturally shaped solid objects

J Farley Norman, Flip Phillips¶, Heather E Ross
Department of Psychology, Western Kentucky University, Bowling Green, KY 42101-3576, USA;
e-mail: Farley.Norman@wku.edu; ¶ Department of Psychology, Skidmore College, Saratoga
Springs, New York, NY 12866-1632, USA
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Abstract. In this study of the informativeness of boundary contours for the perception of natural object shape, observers viewed shadows/silhouettes cast by natural solid objects and were required to adjust the positions of a set of 10 points so that the resulting dotted shape resembled the shape of the original silhouette as closely as possible. For each object, the observers were then asked to indicate the corresponding positions of the 10 points on the original boundary contour. The results showed that there was a close correspondence between the chosen positions of the points and the locations along the boundary contour that were local curvature maxima (convexities or concavities). This finding differs from that of Kennedy and Domander (1985 *Perception* **14** 367 – 370), and shows that, at least for natural objects, the original hypothesis of Attneave (1954 *Psychological Review* **61** 183 – 193) is valid—local curvature maxima are indeed important for the perception of shape.

1 Introduction

The perceptual informativeness of various locations along an object's boundary contour has long been of interest to scientists. For example, nearly 1000 years ago (ca 1030), Ibn al-Haytham (known in the West as Alhazen) stated in his Kitāb al-Manāẓir (ie Optics) that the convexities and concavities of an object's boundary contour played an important role in the perception of its shape. In particular, Ibn al-Haytham presciently wrote (ca 1030/1989, page 172) "for sight will perceive the figure of the surfaces of objects whose parts have different positions by perceiving the convexity, concavity or flatness of those parts, and by perceiving their protuberance or depression". This insight by Ibn al-Haytham was later introduced into European philosophy and science by Roger Bacon (1267/1996). Following the development of psychology, over 600 years later Kurt Koffka (1935, pages 139 – 140) and Fred Attneave (1954) also concluded that the 'dents' and 'protuberances' of an object's outer boundary contour were the most salient parts for the perception of shape.

In the 1950s, Fred Attneave (1954) made the conjectures of the earlier scientists more explicit—he proposed that it was specifically the points of maximum curvature along an object's boundary contour that were the most informative perceptually. He backed up his assertion by including a figure usually referred to as 'Attneave's cat'—it was created by taking the points of maximal curvature from the boundary contour of a cat, and then connecting them with straight lines. The resulting figure was easily recognizable as depicting a cat, and suggested that an economical encoding of at least 2-D shape could be obtained from only the points of maximal curvature themselves. This idea has had a long-lasting impact: for example, Attneave's cat is discussed at length in Regan's (2000) chapter on luminance-defined form. In addition to the presentation of Attneave's cat, in his 1954 paper Attneave also briefly described the results of an empirical experiment he had performed that had shown that the points of maximum curvature along an object's boundary contour were the most perceptually informative for human observers. Surprisingly, however, Attneave never published the complete results of this investigation—there is only the one-paragraph description about it in the theoretically oriented *Psychological Review* article of 1954 (in it, footnote 3 states that the experiment was only published as a "mimeographed research note").

About 20 years after Attneave's seminal contribution, Koenderink and van Doorn (1976; also see Koenderink 1984; Richards et al 1987) showed that the areas of maximal curvature discussed by Attneave have geometrical significance if the 2-D figure is viewed as the projection of a 3-D solid object. In particular, Koenderink and van Doorn mathematically proved that the convexities of a 2-D boundary contour correspond to regions of positive Gaussian curvature, or bump-like regions, on the surface of a 3-D object, while concavities correspond to regions of negative Gaussian curvature, or saddle-like surface regions. This was an important finding, since all smoothly curved solid objects can be represented in terms of these two qualitatively distinct types of surface regions, along with the cylindrical surface regions that divide them (Hilbert and Cohn-Vossen 1952; Koenderink and van Doorn 1978, 1982; Richards et al 1987)

Koenderink and van Doorn's (1976; Koenderink 1984) mathematical analysis demonstrates that some parts of a solid object's boundary contour (ie convexities and concavities) are especially informative about its qualitative shape. The preliminary research of Attneave (1954) suggested that these same regions of the contour were also the most perceptually informative for human observers. It is important to point out, however, that there have been other researchers (eg Kennedy and Domander 1985), who have questioned the validity of Attneave's conclusions. Instead, they have suggested that the areas of "least curvature" (or possibly areas halfway between regions of minimum and maximum curvature) are the most important regions of an object's boundary contour. Our purpose in the present investigation was twofold. One purpose was to resolve this discrepancy by performing an experiment similar to that described by Attneave, since his actual psychophysical results were never described and published in a complete form. An important second purpose was to investigate the informativeness of the boundary contours of naturally shaped 3-D objects. Attneave's unpublished experiment had been concerned solely with 2-D shape, and his contours were apparently generated according to some unknown computational algorithm. If we find results similar to those described by Attneave for natural solid objects, then that would suggest that his conclusions have ecological validity.

2 Method

2.1 *Apparatus*

Silhouettes of natural objects [sweet potatoes (*Ipomoea batatas*)] were created by casting shadows of the potatoes onto a projection screen. The shadows were cast by a 410 W halogen light bulb onto the projection screen, located at a 1.5 m distance from the light source. The silhouettes were then captured with a Toshiba PDR-M5 digital camera and transferred to an Apple Power Macintosh G3 computer.

2.2 *Stimulus displays*

The resolution of the silhouette images of the 12 sweet potatoes as captured by the digital camera was 1600×1200 pixels. The silhouettes were then printed by the Power Macintosh computer onto paper (3 silhouettes per page) with an HP LaserJet 4000N laser printer (600 dpi printer resolution). The final size of the individual rendered silhouettes, on average, was 8.1 cm (horizontal dimension) $\times$ 4.0 cm (vertical dimension). Assuming an average viewing distance of 45 cm, the projected size of the silhouettes was approximately 10.3 deg $\times$ 5.1 deg. The 12 silhouettes as used in the experiment are shown in figure 1.

2.3 *Procedure*

For each of the 12 images, the observers were asked to look at the silhouette and draw, or 'copy', its shape in an adjacent blank area. The observers were instructed to

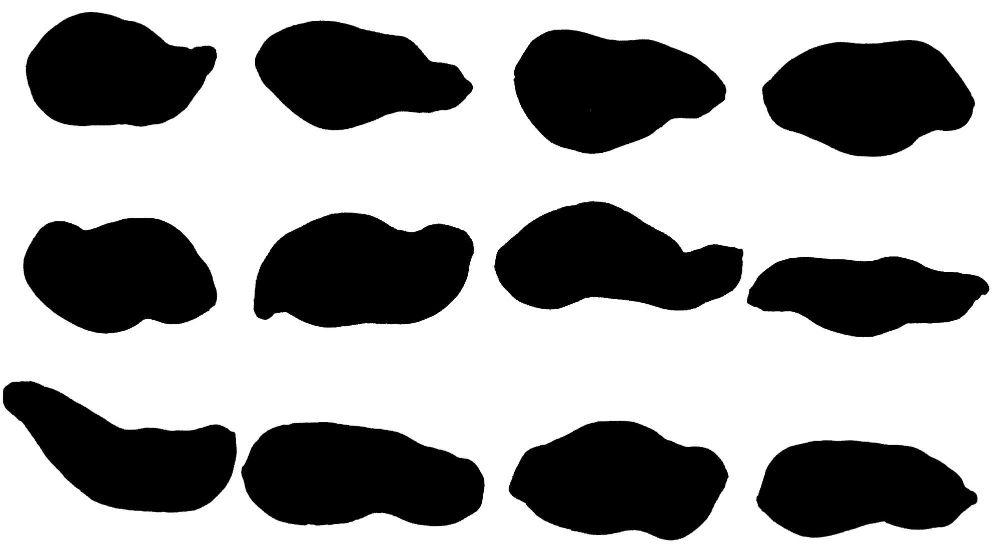

Figure 1. An illustration of the cast shadows of the 12 natural objects used in the experiment.

'copy' the depicted object by positioning (and repositioning, if necessary) 10 points until the dotted figure resembled the original silhouette in shape as closely as possible. When the observers were satisfied with (ie finished adjusting) their drawing, they were then asked to indicate the locations of the 10 corresponding points on the original silhouette boundary contour. In this manner, we were able to determine which parts of the contour were important in copying, and thus perceiving, the shape of the silhouette. By limiting the observers to 10 points, this forced the observers to be 'economical' and pick only those points that were the most important for the perception of shape.

2.4 *Observers*

The observers were twelve undergraduate students, graduate students, and faculty members at Western Kentucky University. All had normal or corrected-to-normal vision. All observers were naïve, in that they were unaware of the purposes and rationale of the experiment.

3 Results

The results of the experiment are shown in figures 2a and 2b for objects 1 – 6 and 7 – 12, respectively. The outer boundary contour of each silhouette was divided into 5 mm segments, and the number of observers who indicated that any given region was perceptually important (by placing one of their 10 points there) is plotted directly adjacent to that portion of the contour. An inspection of these histograms clearly reveals that the observers were not picking regions along the contour at random. Rather, the observers were surprisingly consistent in selecting the same contour regions. A χ^2 analysis was performed for each object to show that the observers were not selecting regions along the contour at random. The χ^2s for each of the 12 objects are shown in figure 3 (all $p < 0.01$).

A careful inspection of the raw histograms shown in figures 2a and 2b would seem to indicate that Attneave (1954) was correct, and that his conclusions also apply to the perception of natural object shape. For every one of the 12 objects, the most frequently chosen parts of the contour were very close to apparent maxima of curvature, either convex maxima (bumps) or concave maxima (dimples). In order to test Attneave's hypothesis more exactly, we calculated the curvatures along the silhouette boundaries.

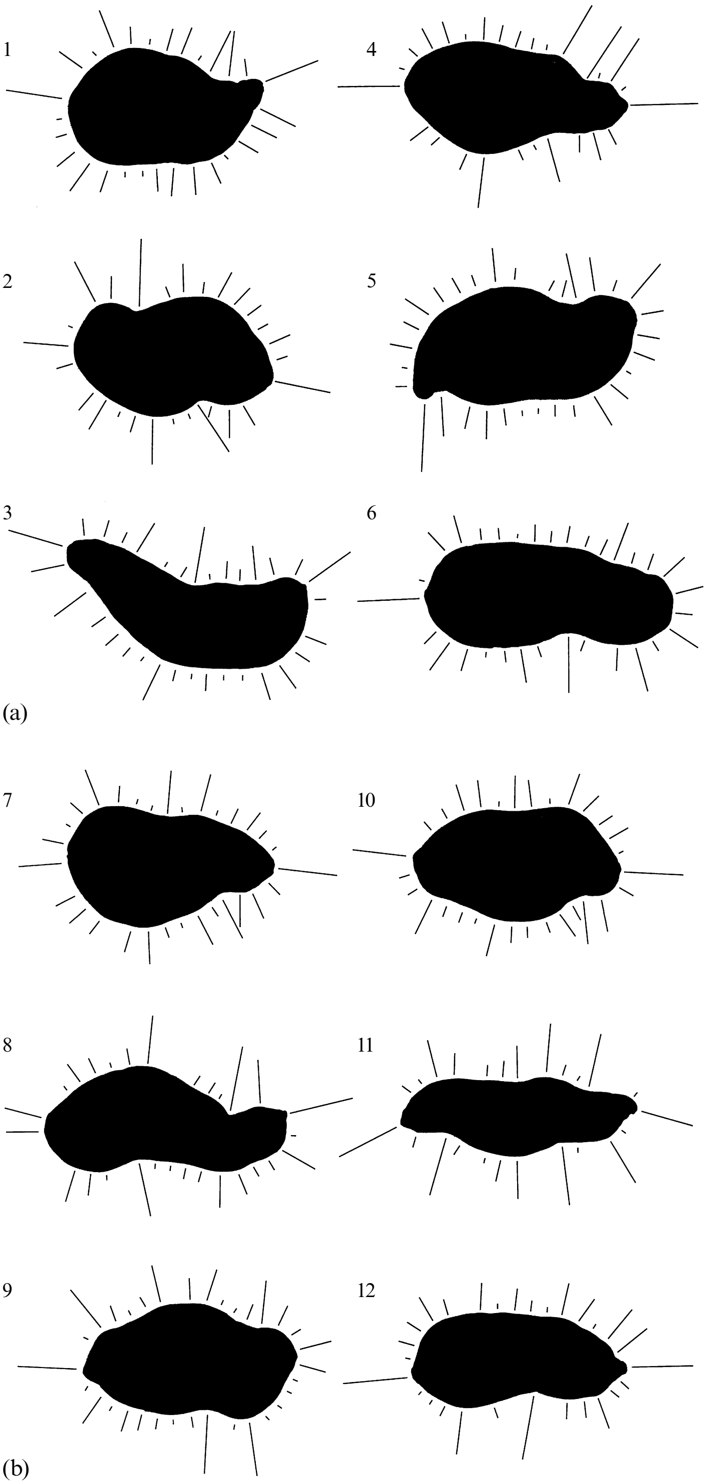

Figure 2. Results of all twelve observers, plotted as histograms. The length of each line segment is proportional to the number of observers who chose positions of points that correspond to each part of the contour when performing the adjustment task.

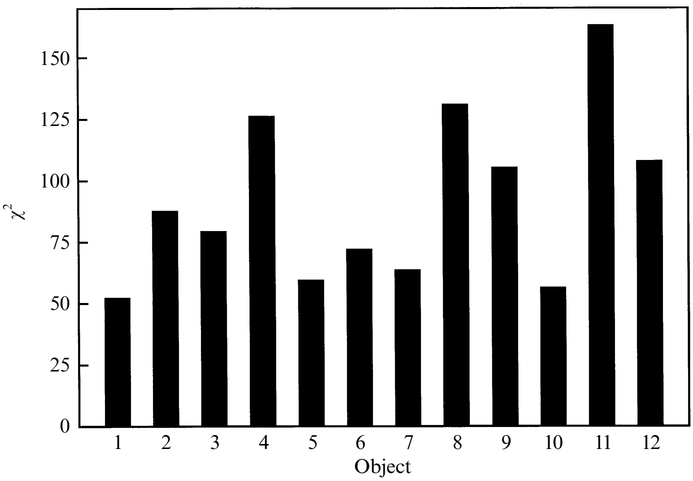

Figure 3. Results of the χ^2 analysis for each of the 12 experimental objects, indicating that the observers were not selecting regions along the contour randomly (all $p < 0.01$).

It is important, though, in performing such an analysis to calculate curvatures at an appropriate scale. In the current experiment, the objects' shadows were digitized at an exceptionally high spatial resolution, resulting in a large amount of structural detail being available at a very small scale. Using this extremely small detail for the purposes of computing the local curvatures would certainly result in extrema that, while mathematically present, could not be detectable by a human observer.

In previous related work, we have been able to determine the threshold amount of structure necessary to detect certain classes of curvatures on 3-D surfaces (Phillips et al 1997). Mathematically, curvature is computed in a strictly local way: the related structural context of the curve has no impact on the local measure of curvature. For example, the surface of a golf ball is almost everywhere concave from a local point of view, due to the dimples used to induce turbulence for longer flight. Globally, on the other hand, the ball is surely a convex object. This phenomenal versus mathematical discrepancy needs to be handled with care if the results of the mathematical analysis are to be sensible from a perceptual standpoint.

In the previous 3-D shape experiments, Phillips et al (1997) computed a 'quasi-local' curvature operator that existed at a variety of spatial scales in an attempt to find the mathematical bounds of the perceptual phenomena of 'bump', 'dent', 'trough', 'saddle', etc. It is a simple extension of these results to apply the same sort of scale-based measure to the 2-D stimuli used in the current experiment.

The actual, discrete, pixel-edges of the shadow images used in this experiment were extracted by using a simple edge-walking algorithm. This resulted in a chain of 2-D positions that exist only on the border of the object. An interpolating piecewise cubic function was fit through this chain of points to provide an appropriate analytical definition of the edge. This function was then convolved with a smoothing filter, the parameters of which were based on the previously described threshold experiments (Phillips et al 1997). For this experiment, the filter was a symmetric Gaussian—the ± 3 standard deviation points were located within approximately 100 edge-points in width. The resulting curvature maxima were then calculated for each shadow image by using this smoothed version of the discrete profile.

Example plots of curvature as a function of distance along the object boundaries are shown in figure 4 for objects 2, 4, and 6. One can clearly see that the curvature plots for these 3 differently shaped objects are indeed quite different. Object 6 (top) is characterized by many small-to-moderate curvatures in both directions, while the

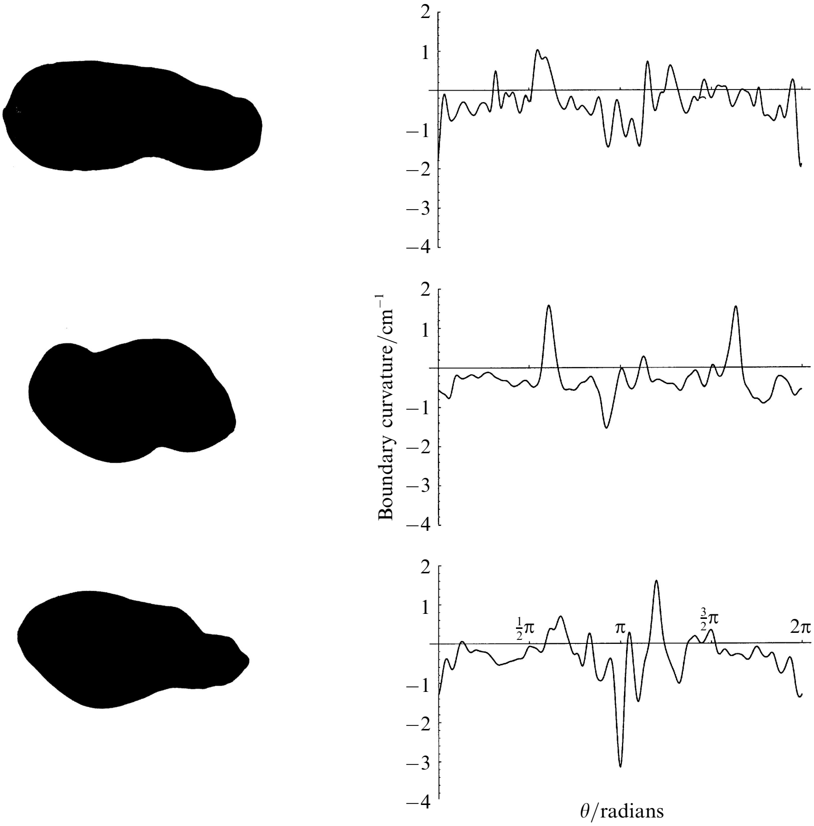

Figure 4. Results of the curvature analysis for 3 representative objects (object 2, middle; object 4, bottom; object 6, top). On the left is the boundary contour itself; on the right the curvature of the boundary is plotted as a function of the angular position, θ, on the contour. Positive curvatures in these plots indicate concavities, while negative curvatures indicate convexities.

curvature plot for object 2 (middle) is characterized by two prominent peaks, reflecting the presence of two salient concavities at the top left and bottom right parts of its silhouette. The curvature plot for object 4 shows one prominent peak and one prominent trough, indicating that object 4 (bottom) has one highly curved convexity (far right of its silhouette) and one highly curved concavity (upper right part of the silhouette).

Curvature plots similar to those shown in figure 4 were derived for all of the 12 objects. The results of this analysis are shown in figures 5a and 5b for objects 1 – 6 and 7 – 12, respectively. The top 16 maxima of curvature of either variety (convexities or concavities) are plotted as filled and open circles. Regions along the contour that were picked by half or more of the observers (ie those regions where there was the most agreement across observers) are indicated by arrows. Notice that for all 12 objects there is a fairly close correspondence between the actual maxima of curvature and the observers' responses. If one looks at any particular arrow, with few exceptions a local curvature maxima will be located nearby. To verify this explicitly, we performed a Wilcoxon matched-pairs signed-ranks test, and for each object compared the number of arrows that were very close (within 5 mm) to a local curvature maxima to the number of arrows that were not close (not within 5 mm) to any nearby local curvature maxima. The results were highly significant ($T_{12} = 0$, $p < 0.01$) and showed that, in performing the shape-copying task, the observers were indeed choosing points located very near to the objects' actual curvature maxima.

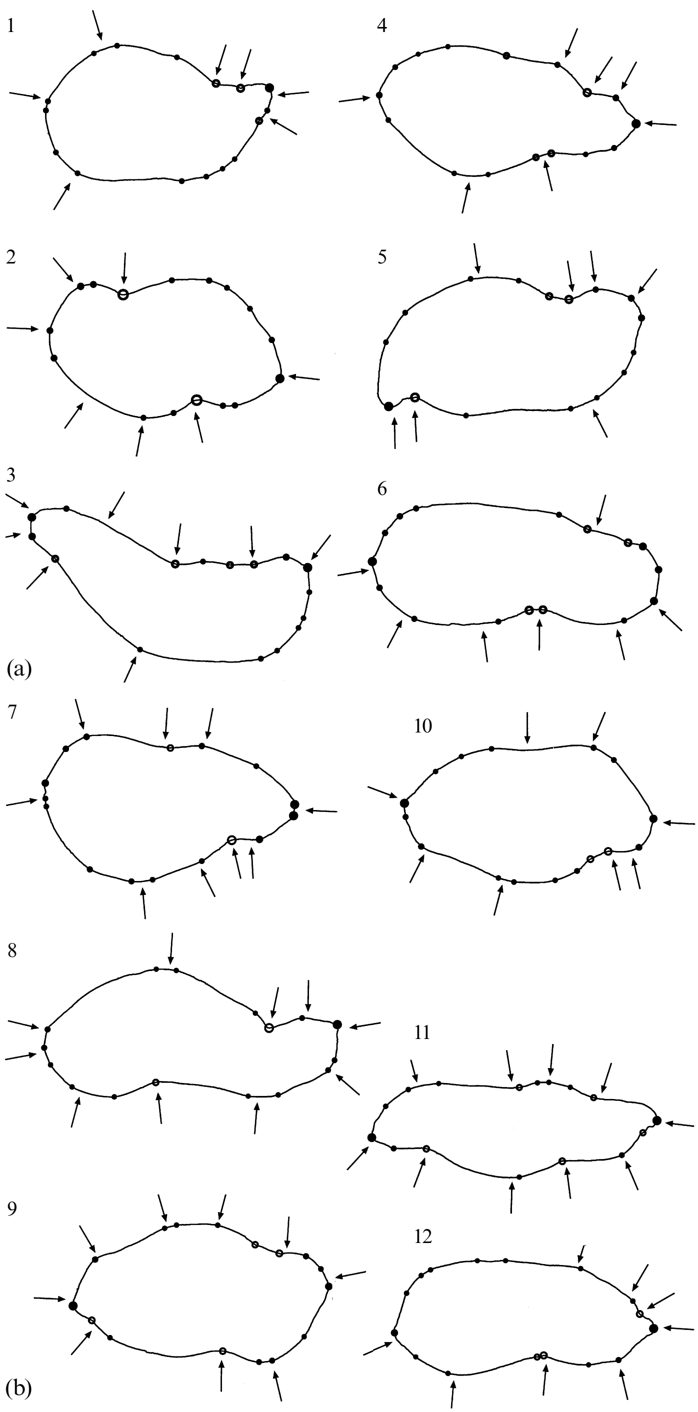

Figure 5. A composite illustration for all 12 objects, showing the locations along the contours that half or more of the observers selected while performing the task (indicated by arrows). The top 16 curvature maxima (of either variety, concave or convex) are also plotted on the contours. The open circles indicate the locations of concavities, while the filled circles indicate the locations of convexities. The size of the circular symbols is proportional to the magnitude of the local curvature maxima, ie the largest symbol for each object represents the local maxima with the greatest amount of curvature, while the smallest symbol represents the local maxima with the least amount of curvature (out of those 16 curvature maxima plotted).

4 Discussion

Mach (1897/1959) pointed out that curved contours are special, in that they easily elicit the perception of a 3-D surface. He also indicated that linear contours can lead to the perception of a 3-D object, but this does not always occur, and whether or not it does occur depends upon other factors, such as the angle of intersection between adjacent lines. In 1913, Bühler (as cited by Graham 1965) found that human observers are exquisitely sensitive at detecting small differences in the curvature of a contour, and found Weber fractions to be 2.5% or less, depending upon specific conditions (also see Watt and Andrews 1982; Treisman and Gormican 1988). In addition, Gibson's (1933) finding of a negative aftereffect of curvature suggests that there are human visual/physiological mechanisms whose function is to detect curvature. The findings of these researchers, such as Bühler and Gibson, suggest that humans have the necessary visual apparatus to detect the curved regions that Attneave (1954) proposed were important for the perception of shape.

Other researchers have proposed specific models for how cortical neurophysiological mechanisms could extract the curvature of contours in a visual stimulus (Hubel and Wiesel 1965; Dobbins et al 1987, 1989; Koenderink and van Doorn 1987; Koenderink and Richards 1988; Kramer and Fahle 1996; Pettet 1999). In addition, it appears that neurons exist within cat and macaque monkey visual cortex that are selectively tuned for the curvature of edges (Dobbins et al 1987; Connor and Pasupathy 1998; Pasupathy and Connor 1999). It is thus likely that similar neurons exist within human visual cortex as well.

Taken as a whole, our results support the informal observations of Ibn al-Haytham (ca 1030/1989), Bacon (1267/1996), Koffka (1935), and Marr (1982) that the convexities and concavities of a boundary contour are important for the perception of shape. Our results are also consistent with Attneave's (1954) conclusion that the specific informative parts of boundary contours are the curvature maxima. Our observers consistently chose regions of locally high curvature when copying the shapes of the silhouettes. There did not appear to be any obvious differences between the perception of convex and concave regions—both were frequently chosen by the observers. As can be seen from an inspection of figures 2a and 2b, observers rarely chose straight (or flat) parts of the silhouette contours—for example, see the results for objects 3 and 6 in figure 2a. Those objects have some relatively long straight contours (eg the top contour of object 6), and those straight regions were rarely selected by the observers. In this respect, our results are different from those of Kennedy and Domander (1985), who found that the straight parts of their contours were the most important for recognition. One possible reason for the discrepancy between our results and those of Kennedy and Domander is that their stimuli depicted rectilinear man-made objects (eg electric clothes dryer, chest of drawers, window, box, etc) with rectangular polygonal faces composed of straight edges (the only 'curved' regions were small, sharp corners where the straight edges intersected), whereas our results were obtained for objects with natural shapes that were characterized by smoothly varying convex and concave curvatures. There were other obvious differences between the two studies as well: for example, there were large differences in the task that the observers performed. The observers of Kennedy and Domander were required to recognize rectilinear objects where various contour regions were occluded from view, whereas in our study the observers could see the entire boundary contour of the objects (ie no occlusion) and were required to adjust the shape of one object to match that of another.

If human observers are sensitive to curvature maxima (convexities/concavities, etc), why do we possess such sensitivity? There are several possible answers. First, it appears that concave maxima along boundary contours have an important role in determining how human observers parse an object into meaningful parts (eg Hoffman and Richards

1984; Singh et al 1999). The analyses of Koenderink and van Doorn (1976; also see Koenderink 1984, 1990; Richards et al 1987) suggest that curvature maxima have a second role. In particular, Koenderink and van Doorn have demonstrated that the regions of significant curvature in a 2-D projection (like a silhouette similar to those used in the present study, or the outline of a projected retinal image of an environmental solid object) have geometrical significance. Convexities in a 2-D boundary contour are the projections of regions of positive Gaussian curvature (bumps) on the surface of a 3-D object, while concavities are the projections of regions of negative Gaussian curvature (saddle-shaped regions). One can thus determine much about an object's qualitative solid shape from an evaluation of the convexities and concavities of its projected retinal image, since areas of positive and negative Gaussian curvature are the only generic types of surface regions on an arbitrary, curved, globally convex 3-D object. In addition, if an environmental object rotates with respect to an observer, because of the rotation in 3-D, previously invisible regions on the surface of the object will ultimately project to the boundary (see Norman et al 2000). If the object rotates through a full 360°, then the observer can form a complete qualitative representation of its solid shape solely from the convexities and concavities of its deforming (ie changing) boundary contour. Giblin and Weiss (1987), Blake and Cipolla (1991), Cipolla and Blake (1992), and Pollick et al (1992) have also shown how to recover shape and curvature from deforming boundary contours.

Given the richness of the information present in the outer boundary contour of an object, it is not surprising that artificial-intelligence researchers and computer scientists have already begun to develop algorithms by which an artificial visual system can extract and recognize the shape of environmental objects from the maximally curved parts of their outer boundary contours (see Chien and Aggarwal 1989; Han and Jang 1990). One reason such a boundary-based approach to recognition is desirable is that these algorithms (tested with real images captured by a limited-resolution video camera) can determine and recognize object shape in the presence of varying illumination, partial occlusion, and varying object orientation relative to the observer (or camera), even in the presence of high amounts of noise. This invariance of recognition performance under changes in illumination, occlusion, noise, etc, is a very desirable characteristic of any visual system, and our human visual system presumably evolved the ability to perceive shape from the curvature maxima of boundary contours for similar reasons and a similar need—to extract and recognize shape in real (ie noisy) environments.

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