Renaissance Teacher Work Sample Consortium

A Teacher Work Sample Exemplar

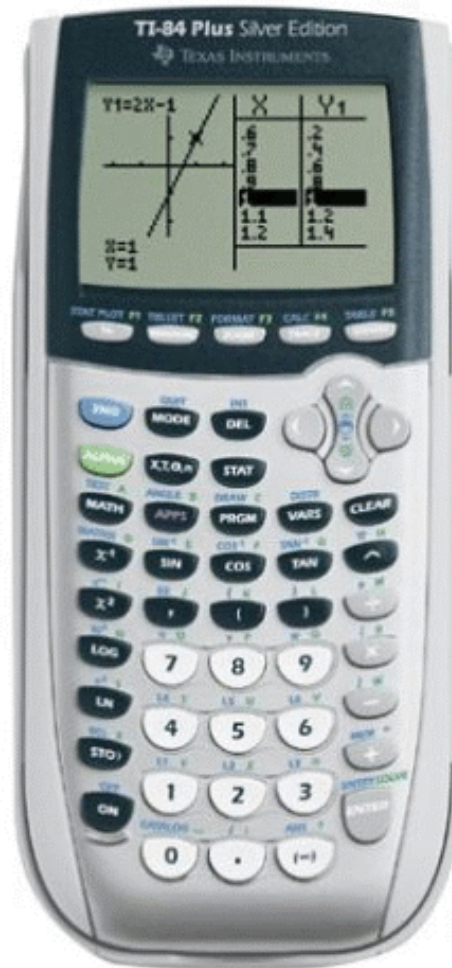Submitted by: Western Washington University

Grade Level: 8ᵗʰ

Subject: Mathematics

Topic: Graphing Calculator

Teacher Work Sample
Grade 8: Graphing Calculator Unit
(Within the Context of Exponential and Linear Relationships)

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CONTEXTUAL FACTORS

Information about the learning-teaching context and student individual differences to set learning goals and plan instruction and assessment

Community, District, and School Factors

Tucked away at the end of a busy street, City Central Elementary is a K-8 school located right next to the district high school campus. Enrollment at City Central is approximately 438 students with 56.2% being female and 43.8% being male. As the fourth largest city in Upland County, this community boasts both commercial growth and fertile farming land. The ethnic makeup of City Central is uniquely composed of 71.4% Hispanic, 23% Caucasian, 0.9% African American, 0.9% American Indian, and 0.2% Asian or Pacific Islander students.

One of City Central's strongest programs (inspired by the prevalence of Hispanic students) is the "Two-Way Enrichment Dual Language (Spanish/English) Program" that extends into the fourth grade this year. Students in bilingual classrooms are grouped into pairs of one "English Expert" and one "Spanish Expert." The pairs work together to learn content presented in English (mathematics), content presented in Spanish (social studies and science), and literacy in their expert languages. Bilingual abilities are highly valued, supported, celebrated, and grown at this school. Walking into the main office at City Central, one is more likely to hear Jill (the head secretary) speaking on the telephone to a parent in Spanish than English. Every handout, permission slip, and announcement board boast both Spanish and English words side-by-side.

Classroom Factors

Within this K-8 school exists a separate community of students… the "middle-schoolers" who inhabit their very own set of portables behind the main K-6 building. Rotating between language arts, social studies, science, mathematics, AVID (a college preparation program), school-to-work (a life skills course), and PE, these seventh and eighth grade students operate according to a six period

schedule. Students failing a class for one quarter attend a seventh period the following quarter in order to receive extra academic support.

The classroom in which the Teacher Work Sample (TWS) will take place is Mrs. Sally Mountain's mathematics classroom (during 4ᵗʰ and 6ᵗʰ period). At this time, Mrs. Mountain's room solely hosts eighth-grade students. Tables are arranged in groups that each host 4-6 students. These groups are static, so that students know their learning partners well. The following are the posted, student-generated group work expectations: no put downs; share the work; keep "you to you;" use group roles; listen; and maintain on-task and in-group conversations only. Just this year, Mrs. Mountain received a grant that enabled her to purchase a class set of TI-84 graphing calculators daily accessible by students. The classroom is also equipped with a document camera and two whiteboards (one for instruction and one for assignment postings).

Student Characteristics
Approximately 67% of the eighth-grade students in sixth period (my focus class) are Hispanic, with 4 receiving ELL services (but many more with Spanish as their L1), 2 students receiving special education services, and 6 students receiving Title I mathematics services. One special education assistant and one Title I specialist is present in sixth period every day. Extra professional personnel in the class provide students with extra academic support and enable teachers to collaborate in formative assessment of student learning. It is important to note that parents are not present in the classroom setting. Although some parents help their students with homework at home, many parents do not have a well-developed education that would enable them to be academically supportive of mathematics content.

These eighth-graders have received little instruction on calculator usage this year (apart from how to enter basic operations, clear the memory, and turn

them on/off). They have never participated in a lesson focused upon graphing calculator usage alone. Students are accustomed to working on computers in their social studies class where each student has a computer in front of them for the entire period and a Smart Board enriches their teacher's instruction. However, poverty (indicated by City Central's 78.5% Free/Reduced Lunch Count) has limited many students' educational technology experiences at home. For most, school is the only place where they can engage in technological activities that enhance mathematical understanding.

Instructional Implications

The bilingual background of many of my students will impact my planning in several ways. Primarily, I must keep my collegiate mathematics talk in balance with explicit vocabulary instruction. Students may have mastered conversational English, but require extra support in regards to mathematics-specific terms (via visuals and Spanish translations of some handout components). The entire unit stresses approaching the two concepts (and vocabulary terms) of exponential and linear relationships in multiple formats of equations, tables, and graphs. "Seeing" these relationships in multiple formats will attend to the needs of bilingual students to encounter concepts in various representations.

Because these eighth grade-students lack graphing calculator experience, I will be challenged to draw upon other "non-calculator" background knowledge and make relevant connections. The students have extensive practice creating tables and graphs by hand of linear equations, and have worked with exponential equations for three weeks preceding the mini unit. I will refer to familiar linear equations and exponential relationship problems with which the students have worked. Additionally, I will endeavor to establish clear, convincing reasons for using calculators so that students see the value in and purpose of graphing calculator activities (to enrich learning, not replace thinking). Since students have utilized the

powerful tool of a graphing calculator for mere standard operations all year long, I will seek to provide a profound introduction to the impressive, meaningful possibilities of technology in the mathematics classroom!

Classroom routines, such as daily group work within the structure of group roles, will be maintained in my own lessons. Because my lessons are separate from (enriching) the district curriculum (Connected Mathematics II), I will have more freedom in lesson structure. However, I believe that consistency with CMPII instruction structure (launch, explore, summarize sections) will support student learning. As much as possible, I will utilize the extra professional support present in my classroom and strive to create a learning experience that is culturally aware and specifically formulated for these eighth-grade students.

LEARNING GOALS
Setting significant, challenging, varied and appropriate learning goals

This mini-unit will be taught to two sets of eighth-grade students, one fourth period class and one sixth period class (each period lasting for 50 minutes). Although the students have used a brand new set of graphing calculators for simple calculations all year long, this will be the first opportunity for many to use its specialized functions (creating tables and graphs). The aim of the mini-lesson will be for students to develop technical graphing calculator skills within the context of analyzing the characteristics of tables and graphs that represent exponential and linear relationships. The following essential question (EQ) will guide this lesson: *How and why do we use graphing calculators in the context of understanding exponential and linear relationships?*

Students will have answered the EQ when they meet the two mini-unit learning goals:

1. Within the context of variable relationships (exponential and linear), students will **use** equations to create a *table* of values on a graphing calculator that they will **analyze** in order to identify characteristics of the given variable relationships. (LG1)

   - Outcome A: Students will **use** one exponential and one linear equation to create a table of values on a graphing calculator.

   - Outcome B: Students will **analyze** a table of values from one exponential and one linear equation in order to identify the characteristics of exponential and linear relationships as they appear in tables.

2. Within the context of variable relationships (exponential and linear), students will **use** equations to create a *graph* on a graphing calculator that they will **analyze** in order to identify characteristics of the given variable relationships. (LG2)

   - Outcome A: Students will **use** one exponential and one linear equation to create a graph on a graphing calculator.

   - Outcome B: Students will **analyze** a graph of one exponential and one linear equation in order to identify the characteristics of exponential and linear relationships as they appear in graphs.

The eighth-grade graphing calculator unit aligns with Washington state's mathematics **EALR 1. The student understands and applies the concepts and procedures of mathematics.** The unit aligns specifically to the 8th grade **GLE 1.5.1 Apply knowledge of linear and non-linear relationships to recognize, extend, and create patterns and sequences in tables and graphs.** LG1 addresses the application of relationship knowledge to recognize patterns in *tables*, while LG1 addresses the same application of knowledge to recognize patterns in *graphs*. OSPI offers the following example objective in correlation with GLE 1.1.5: **Students will use technology to generate graphic representations of linear and non-linear relationships.** Therefore, the integration of technology (graphing calculators) into this unit matches OSPI's interpretation of GLE 1.1.5.

A student who successfully meets LG1 will be able to create and analyze table representations of exponential and linear relationships. The ability to see multiple representations of variable relationships while constructing mathematical solutions will open the door for many students to answer conceptual questions without getting lost in graph construction "by hand". Van de Walle also recognizes the freedom that follows from application of calculator strategies. He suggests that,

"many excellent explorations that happen spontaneously in a problem-solving environment will be enhanced by the use of calculators" (2007).

A student who successfully meets LG2 will be able to create and analyze graphical representations of exponential and linear relationships. The graphical representation is one of the clearest ways to analyze variable relationships. Students will use their table-making skills in order to inform their decisions about adjusting window settings to construct meaningful graphs (connection between LG1 and LG2).

The learning goals of the graphing calculator mini-unit were crafted to initiate various levels of thought in students (according to Bloom's Taxonomy). LG1 and LG2 meet the **application** level of Bloom's Taxonomy by expecting students to apply their skills of creating tables and graphs on graphing calculators. LG1 and LG2 also meet the **evaluation** level (highest level) of Bloom's Taxonomy by expecting students to analyze the two representations (connecting the patterns of growth they observe to the exponential and linear relationships they are coming to understand).

Both learning goals are appropriate in terms of student development because they will be seamlessly connected to the curriculum portion (CMP2 *Growing, Growing, Growing*) approached during the month leading up to the mini-unit. Student interest in using the impressive graphing calculators in the back of the room is high, and replacement of graphing "by hand" by graphing with calculators will enable many students with special needs to focus on mathematical concepts without the barrier of pencil and paper methods. Students who choose to fully engage in the unit will reap the ability (by way of the graphing calculator technology) to see exponential and linear relationships through the three representations of equations, tables, and graphs.

ASSESSMENT PLAN
Using multiple assessment modes and approaches aligned with learning goals to assess student learning before, during, and after instruction.

Overview Tables

* Note: Analysis of assessment will be based upon a sample group of five students from 6th period.

Learning Goals	Assessments	Format of Assessment	Adaptations
Learning Goal 1: Students will choose two equations (one representing an exponential relationship and one representing a linear relationship) and **use** them to create a table of values on a graphing calculator. Students will then **analyze** the table in order to identify the characteristics of exponential and linear relationships as they appear in tables.	Pre-Assessment	Lesson one warm-up written response (choosing exponential and linear equations)	Fill in the blank (cloze) statements will guide student responses. Equations presented will be familiar to students (come from previous linear and exponential activities).
	Formative Assessment	• Think-pair-share class discussion regarding table characteristics of exponential and linear relationships • Specific CFU questions pertaining to meaning of values in table • Anecdotal notes on students' table-making strengths/needs and identification of characteristics of relationships (during several table making learning activities – CMP Problem 2.1 and 2.2) • Review after class of written student responses on p.5 of student packet (Table Representations of Exponential and Linear Relationships) (See Appendix D)	Each student will have a "Table Reminder Card" to assist him/her in remembering the steps in creating a table on a graphing calculator. Think-pair-share prompt will be written on the board as well as spoken aloud.
	Post-Assessment	Performance-based task/personal interview on graphing calculator table-making and analysis (to	Students will be welcome to use their graphic organizer chart

		identify linear and exponential variable relationships)	"Characteristics of Relationships Between Variables" and/or "Table Reminder Card"

Learning Goal 2: Students will **use** two equations (one representing an exponential relationship and one representing a linear relationship) from the previous lesson to create a graph on a graphing calculator. Students will then **analyze** the graphs in order to identify the characteristics of exponential and linear relationships as they appear in graphs.	Pre-Assessment	Anecdotal notes on students' written responses and verbal remarks during "launch" part of lesson #2 in regards to Problem 2.1, question B (See Appendix E)	Problem 2.1, question B asks students to use prior experience with exponential graphs (investigation 1) to analyze a new graph. They will be encouraged to look at page 8 (use math book as a resource) during class discussion.
	Formative Assessment	• Review of students' written responses in completion of Problem 2.1 and 2.2 (part of Investigation 2 homework packet) • Anecdotal notes on students' graph-making strengths/needs and identification of characteristics of relationships (during several graph-making learning activities – CMP Problem 2.1 and 2.2)	Each student will have a "Graph Reminder Card" to assist him/her in remembering the steps in creating a graph on a graphing calculator. Group task instructions will be given verbally and listed on the whiteboard.
	Post-Assessment	Performance-based task/personal interview on graphing calculator graph-making and analysis (to identify linear and exponential variable relationships)	Students will be welcome to use their graphic organizer chart "Characteristics of Relationships Between Variables" and/or "Table Reminder Card"

Pre-Assessment

At the very beginning of the first lesson, students will complete a pre-assessment activity that requires them to identify an exponential and a linear equation from a list of familiar equations and provide a rational for their choices (See page 2 of student packet, Appendix D). Students will be graded according to the ERR (See Appendix A) that assigns up to one point for each equation and up to three points for each rationale statement. Erased equations (if correct) will be given full points because I want to give credit to students with hesitancies and/or low mathematical confidence. Rationale statements may be "strong" (3 points for a detailed, correct, and possibly vocabulary rich response), "sufficient" (2 points for a correct response), "weak" (1 point for a response that could be redirected to correctness), or "non-existent/wrong" (0 points for a blank space or incorrect response). My overall goal will be to score the pre-assessment and then highlight high, middle, and low students in various colors so that I can have a quick visual for where students are at in their reasoning for variable relationships.

I chose to pre-assess students' concepts of variable relationships in the *equation* representation (as opposed to the table and graph representations) because the students have the most experience drawing connections between equations and linear or exponential relationships. I felt that this representation might draw out the most detailed (and vocabulary rich) rationales for their responses. Starting with the most familiar representation of equations also might draw out common misconceptions that I will need to address in regards to the other representations (LG1 and LG2).

Post-Assessment

Due to the complexity of assessing the *skills* of creating tables and graphs on a graphing calculator alongside assessing student *analysis* of those tables and graphs (LG1 and LG2), I will conduct a performance-based task for my focus group of students to complete during personal one-on-one interviews. Each student will be given a sheet of paper with task instructions. I will read the instructions aloud as they follow along. Students will have access to both the "Characteristics of Relationships Between Variables" chart and the "Calculator Table and Graph Reminder Cards." Students will be introduced to a real-life linear relationship and then an exponential relationship and asked to do the following for each:

1. Identify whether the situation is linear or exponential.

2. Write an equation to represent the relationship.

3. Create a table of values for the relationship and provide one reason for why the table represents a linear/exponential relationship.

4. Create a graph of the relationship and provide one reason for why the graph represents a linear/exponential relationship.

Because student success at each step is dependent upon success with the previous step, I will allow wait time and then give correct responses and appropriate explanations. In this way, I will be able to assess each component of LG1 and LG2 independently. My rubric for scoring the student interview (See Appendix C) is a combination of checklists and anecdotal notes that correspond directly to the two learning goals and complement the ERR from the pre-assessment (by including assessment of skills *and* rationale).

Formative Assessment

Assessment of students' table and graph making skills will be addressed through an anecdotal note-taking system where I can write down specific need areas in regard to graphing calculator operation on my FAAN chart (See Appendix

B). This chart will also be a place for me to record characteristics that students have identified (or lack of progress). I will review student packets and students' written work on CMP problems 2.1 and 2.2, adding notes to the FAAN chart according to these informal observations. Additionally, I have built into the unit specific questions to ask students to check for understanding (CFU). Their responses will determine whether or not we move on and which extra explanations I present during instruction.

I will rely upon other teachers and assistants in the room (contextual factors) in order to formatively assess students' progress in table and graph making. At the end of every lesson, I will ask them to identify specific students and needs that they noticed while assisting individuals. This information will be recorded in my FAAN chart and inform my instruction in following lessons.

DESIGN FOR INSTRUCTION
Designing instruction for specific learning goals, student characteristics and needs, and learning contexts

Results from Pre-Assessment

My choice of the pre-assessment task of students identifying linear and exponential relationships in *equation* representation stemmed from the students' contextual factors of previously covered content (a whole unit on linear relationships and three weeks of working with exponential equations). It can be more difficult to rationalize variable relationships in the equation representation (as compared to the table or graphical representations). Despite background experiences with the content, many students struggled with this task (confirming the need for the unit).

Consider the data from the pre-assessment (See Table 1 below). I made response rationale more valuable than pure identification of equations. The data suggests that students need to develop solid rationale for identifying variable relationships. Three out of my six target students left rationale spaces blank, while one honestly recorded that she "guessed." The misconception (revealed through the pre-assessment) that variable quantity determines variable relationships will need to be addressed. One student did provide mathematically sound reasoning for her equation choices, but needs to be introduced to formal mathematics vocabulary.

From the data, it is clear to me that students will need:

A.   Instruction on how to identify variable relationships in the equation representation (something that I thought they could identify, but maybe not explain).

B.   Additionally, students need to develop mathematical vocabulary for the concepts that students hinted at in their rationales for choosing linear and exponential equations. For example, students

need to become comfortable with terms like "constant rate of
change" and "growth factor."

I will be sure to re-teach specific skills on how to identify certain variable
relationships by looking at an equation. I will also emphasize specific mathematical
terms (growth rate, slope, y-intercept, start value, and rate of change) in the chart
that we complete as a class to record our learning and throughout the mini unit.

Table 1: Pre-Assessment Results

Student	Warm-up 1 EE, LE, ER, LR, **Total**					Other Notes
V.E.	0	0	0	0	0/8	EE and LE switched, R: variable quantity
E.G.	1	1	1	1	4/8	Erased correct EE, R: guessed
R.G.	1	0	0	0	1/8	Erased correct EE, all else blank
A.H.	0	0	0	0	0/8	Blank
E.H.	0	0	0	0	0/8	Blank
M.T.	1	1	3	3	8/8	LE hints at constant rate of growth "adds 3"

- EE stands for exponential equation. LE stands for linear equation. Students either have the equations (1) or do not (0).
- ER stands for exponential rationale. LR stands for linear rationale. Students may have a strong (3), sufficient (2), weak/misguided (1), or non-existent (0) rationale.

Unit Overview

Learning Activity	Learning Goals	Instructional Activities	Assessments
A	LG1: Students will choose two equations (one representing an exponential relationship and one representing a linear relationship) and use them to create a table of values. Students will then analyze the table in order to identify the characteristics of exponential and linear relationships.	Lesson 1: Constructing tables on the TI-84 Plus (within context of exponential and linear relationships) • Equations warm-up (individual written task) • Introduce "Characteristics of Variable Relationships" chart • Re-teach characteristics of linear/exponential relationships in equation form (based on pre-assessment student needs) • Creating a table mini-lesson • CFU class discussion ADAPTATIONS: • "Characteristics of Variable Relationships" chart formulated as a note-taking graphic organizer • Specific vocabulary terms emphasized in chart • Calculator screen displayed through document camera • Long wait time between table-making steps (with extra classroom teachers checking to see that each student is on track)	Pre-Assessment • Equations warm-up Formative Assessment • Anecdotal notes on table-making • Review student notes in equation row of "Characteristics of Variable Relationships" chart

B	LG1	(Students worked toward reaching LG1 while I was not present) • Group task: "Table Representations of Exponential and Linear Relationships" activity (page 5 in student packet)	Formative Assessment: • Review student responses on page 5/anecdotal notes

C	LG1 and LG2: Students will use two equations (one representing an exponential relationship and one representing a linear relationship) from the previous lesson to create a graph on a graphing calculator. Students will then analyze the graphs in order to identify the characteristics of exponential and linear relationships as they appear in graphs.	Lesson 2: Constructing graphs on the TI-84 Plus (within the context of exponential and linear relationships) • Fill out table row of "Characteristics of Variable Relationships" chart based upon group responses to questions 4 and 5 from page 5 of packet • Getting Ready for Problem 2.1 class discussion • Introduce Problem 2.1: Model/think aloud process of interpreting word problem and translating it into a table and equation • Student work time on Problem 2.1 ADAPTATIONS: • Utilize graphic organizer as note-taking guide • Think aloud while interpreting word problem • Record important information from word problem on white board in organized, color-coded fashion	Formative Assessment • Filling out table row of "Characteristics of Variable Relationships" chart • Anecdotal notes on graph-making and students' written responses to question 2.1

D	LG2	(Students worked toward reaching LG1 while I was not present) • Students began Problem 2.2	Formative Assessment • Review student work on Problem 2.2/anecdotal notes
E	LG1 and LG2	Lesson 3: Applying table and graph making skills on a graphing calculator to Problem 2.2 • Class discussion to fill out graph row of "Characteristics of Variable Relationships" chart • Model/think aloud process of interpreting word problem and translating it into a table and equation • Clarify several common calculator errors (^ symbol and QUIT option) – Identified during formative assessments • Review answers to Problem 2.2 • Student homework work time ADAPTATIONS: • Class schedule and materials needed posted on whiteboard • Utilize graphic organizer as note-taking device • Whiteboard separated into three sections to emphasize three representations • Graphing calculator button symbols recorded on board as appear on calculator	Formative Assessment • Filling out graph row of "Characteristics of Variable Relationships" chart • Anecdotal notes on student responses to Problem 2.2

Summative Assessment	LG1 and LG2	One-on-one interviews ADAPTATIONS: See adaptations portion in assessment plan matrix.	Summative Assessment • Performance-based task evaluated during one-on-one interviews (table/graph making skills and variable relationship reasoning both assessed)

Learning Activities

ACTIVITY 1: Creating a table mini-lesson (part of lesson one)

The mini-lesson on table making gave students their first opportunity to use a graphing calculator beyond basic operations. While I explained and displayed each step of the process, students followed along (entering linear and exponential equations, changing table settings, and reviewing the actual table). All teachers in the classroom checked to see that each student was following along, and I asked specific CFU questions about the meaning of the values in the table. The mini-lesson stemmed from the contextual factor that students had no formal training on how to use a graphing calculator to create tables. The activity addressed outcome A of LG1 (*using* equations to create a table on a graphing calculator). TI-84 Plus graphing calculators and calculator table reminder cards enabled students to reach LG1, while I formatively assessed students in the FAAN chart and made additions according to conversations with the other teachers/assistants in the classroom.

ACTIVITY 2: "Table Representations of Exponential and Linear Relationships" activity (in between lessons one and two)

On the day in between my first and second days of instruction, students worked in groups to complete page five of the student packet (see appendix D). The activity asked them to decide upon one exponential and one linear equation to analyze, create a table of values from the equations, identify what the values in the three

columns represented, and (most importantly) record one characteristic for an exponential relationship and one characteristic for an exponential relationship in table form. The activity stemmed from students' difficulty in identifying linear and exponential equations in my pre-assessment. Asking students to do such in this activity enabled me to formatively assess their progress in identifying linear and exponential equations (a prerequisite skill for reaching LG1 and LG2). The group activity addressed outcome B of LG1 by asking students to *analyze* a table of values and identify characteristics of exponential and linear relationships in table form (questions 4 and 5 on page 5 of the student packet). Students were required to utilize graphing calculators and had access to materials such as note from the previous day's lesson and the calculator table reminder card. Reviewing student responses from this activity proved to be a very informative form of formative assessment. Based upon my observations of their packets, I addressed common misconceptions and strengths throughout my next day of instruction.

ACTIVITY 3: "Modeling the process of interpreting a given problem into equation and table form" (part of lesson three)

A common dilemma that arises for the students is that of interpreting the language of mathematics problems. Throughout my teacher education at WWU, I have read several articles emphasizing that the "entry" level of problem solving is the most important step. Unless students understand what the problem is asking them to do, they cannot make much progress! Student progress on problem 2.2 (See Appendix F), on the day in between my second and third days of instruction, was greatly hindered by the language of the problem. Part of the language troubles are due to the fact that many students speak English as a second language (contextual factor); however, the language in the CMP curriculum requires scaffolding for native English speakers to interpret as well. Therefore, an explicit modeling of interpreting the language of problem 2.2 in a "think aloud" format was extremely relevant. The

process of pulling out important information from the text and identifying what "we know" and what "we want to find out" was put within the context of graphing calculator usage. In using the graphing calculators to answer problem 2.2, students addressed LG1 (outcome A) and LG2 (outcome A). Students again used the technology of graphing calculators and materials such as the student packet, calculator table/graph reminder cards, the "Characteristics of Variable Relationships" chart, and their CMP math books. I formatively assessed students during group work time when I could observe and make notes on my FAAN chart regarding student progress in interpreting the language of problem 2.2.

Technology

Technology use is a vital component of my mini-unit. The entire focus is upon graphing calculator usage within the context of analyzing tables and graphs in order to identify specific variable relationship characteristics. Displaying the teacher calculator on the front screen (using a document camera) in the classroom while students work on their own calculators occurs frequently. The overhead is used to display the "Characteristics of Variable Relationships" chart, and the document camera displays other pages of the student handbook.

INSTRUCTIONAL DECISION-MAKING
Two modifications made to design for instruction based upon student learning or responses

MODIFICATION 1: Re-teach characteristics of linear and exponential relationships in equation representation

Evidence of Need

After reviewing student responses (as recorded on the ERR, Appendix A) on the pre-assessment activity that asked students to identify linear and exponential equations from a list of familiar equations and provide rationale for their decisions, it became apparent that students needed a mini-lesson on the characteristics of linear and exponential relationships in equation representation (before they would be able to deal with the relationships in table and graph representations). The following figure demonstrates the need for equation representation instruction, based upon the high percentage of students that were only able to identify one equation correctly (26%) or were not able to identify either equation correctly (41%). Note that only one-third of the students correctly identified both equations.

Student That Correctly Idenitified Equations

- Linear Only
- Exponential Only
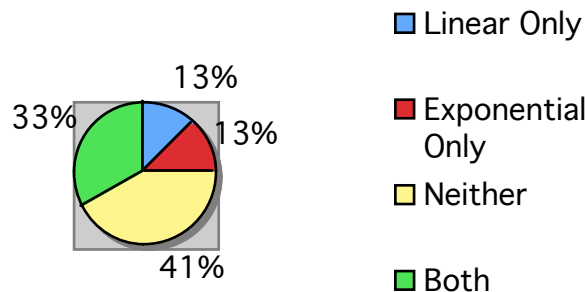- Neither
- Both

13%
13%
33%
41%

Figure 1: Pre-Assessment Data

Evidence of Modification and Effectiveness

Based upon the pre-assessment data above, I modified my first lesson by re-teaching the characteristics of linear and exponential relationships in equation form. I provided explicit notes for students to record in their "Characteristics of Relationships" chart and connected (by way of class discussion) the attributes of slope, y-intercept, growth factor, and start value to specific example situations from the curriculum that students had encountered before. Students needed to recognize linear and exponential relationships in equation form (a pre-requisite skill that I thought they had mastered, but appeared as a need area in the pre-assessment) in order to engage in the unit activities and meet LG1 and LG2.

I know that the re-teach modification was effective because after reviewing students' "Characteristics of Relationships" charts, the majority of students had recorded standard linear and exponential equation forms, labeling specific variables. Additionally, students continually brought up the argument during following class discussions that the reason why a situation was exponential was because, "the equation has an exponent." Students would not have been able to show such success as they did in the unit (see Analysis of Student Learning section), without developing the ability to identify and differentiate between exponential and linear equations (a pre-requisite still for LG1 and LG2). This modification to my design for instruction enabled students to experience such success.

MODIFICATION 2: Provide language support to interpret word problems and translate information into equation, table, and graph representations

Evidence of Need

Language support is a continual need with my student population. In this unit, it became apparent (after a day of students working on CMP's problem 2.1) that I had not provided enough language scaffolding in my design for instruction to enable students to interpret word problems. My cooperating teacher informed me that students were continually asking questions that demonstrated confusion about what the problem 2.1 was asking them to do. She said that I needed to provide them with a modeling experience where I thought aloud while interpreting a word problem and translated the information into equation and table representations.

Evidence of Effectiveness

Based upon my conversation with my cooperating teacher, I modified my second lesson to include such a "think aloud" experience for my students. I read the problem aloud and pulled out the important information, highlighting the strategies of underlining and translating data into table and equation form. I demonstrated how to identify growth factors and start values and use these figures to created tables (from which equations could be formed). After this demonstration, students experienced success with interpreting problem 2.2 later in the unit. I noticed less confusion in regards to the language of the problem and more content-focused questions during cooperative group work time, again enabling students to access the problems leading them towards meeting both learning goals 1 and 2.

ANALYSIS OF STUDENT LEARNING

Use of assessment data to profile student learning and communicate information about student (group and individual) progress and achievement

Group Analysis

Introduction

The five students in my focus group were selected based on the range in their current mathematics performance academically (2 low, 2 medium, 1 high), variation in gender (2 girls, 3 boys), English language proficiency (3 out of 5 have an L1 of Spanish), and other special needs. One of my students has been referred for special education, but not officially assessed or provided with services (although she receives Title I services). Another student receives both ELL and Title I services. The one high student in the group often works at her own pace ahead of the class, sometimes holding onto her misconceptions as she rushes through the content. I chose a diverse focus group in order to assess the effectiveness of my instruction in meeting learning goals in the case of various academic, language, and special needs student circumstances.

Learning Goals

For the sake of narrowing my analysis, I chose to focus upon the following two outcomes from my learning goals:

- Learning Goal 1, Outcome B: Students will **analyze** a table of values from one exponential and one linear equation in order to *identify the characteristics of exponential and linear relationships as they appear in tables*.

- Learning Goal 2, Outcome B: Students will **analyze** a graph of one exponential and one linear equation in order to *identify the characteristics of exponential and linear relationships as they appear in graphs*.

Assessment Data

The heart of LG1 and LG2's second outcomes (Outcome B) is the goal of taking students to the place where they can *identify the characteristics of linear and exponential relationships in various representations* (more specifically, in the table and the graph). In order to get to these two representations, I had to assess students' abilities to recognize the relationships in word problems and equations too. My pre-assessments focused upon assessment of these pre-requisite skills, but still related to "identifying the characteristics of linear and exponential relationships."

To get a foundation for understanding students' level in identifying linear and exponential relationships (and providing justification for such identification), I scored a warm-up activity that had students identify a linear and an exponential equation and equation their choices (see Appendix D for the warm-up activity and Appendix A for the scoring rubric). This activity gave me a basis for student ability to identify variable relationships in general. Later assessments would assess identification of variable relationships in table form (LG1, Outcome B) and in graph form (LG2, Outcome B).

After a pre-assessment, I formatively assessed students' analyses of tables and graphs to identify characteristics of variable relationships by taking anecdotal notes during work time. These notes were not very detailed, but offered a bit of insight into their analyses development (See Appendix H, last column of table).

In terms of post assessment of LG1/2 (Outcomes B), I formulated questions in the interview specifically meant to pinpoint student progress in analyzing tables and graphs. For both situation A and B, students were asked to construct tables and graphs demonstrating a linear and exponential variable relationship (if they had trouble, I prompted and re-taught them). Then students were asked how they knew that the table and graph were exponential or linear. Students' responses were recorded on the interview rubrics under the analyses items (See Appendix C) as a

numerical score. I also added anecdotal notes if students had unique, insightful responses (See Appendix I).

Summary and Evidence (Graphic Representations of Assessment Data)

From a purely pre-assessment standpoint, the students in my focus group demonstrated a high need for instruction on identifying and rationalizing variable relationships. Consider the pre-assessment column of the compilation table in Appendix H where out of four areas of assessment (identification of exponential and linear and rationale for exponential and linear), only one student scored perfectly; one student demonstrated 25% proficiency (and that was in the area of identifying, not rationalizing); and the remaining three students received a score of zero. Of course, I must consider that I was pre-assessing only the equation representation. Students might have scored higher had other representations been available (I would provide graphs and tables in a pre-assessment if I were to redo the unit). Besides this fact, the need for instruction in strategies to identify variable relationships appeared extremely high from a pre-assessment analysis. The pre-assessment data is important because it proved to me that I needed to provide experiences for the students to practice identifying variable relationships before they could begin to provide justifications for *why* representations were linear or exponential (exactly what LG1/2, Outcomes B ask students to demonstrate).

Formative assessments (anecdotal notes) demonstrated student progress towards and misconceptions related to LG1/2, Outcomes B. Note the formative notes column of the table in Appendix H. Progress towards analysis of tables (LG1 Outcome B) can be see in Maroon's "identification of the growth factor of an exponential relationship in table representation." When students began to recognize start value, growth factor, y-intercept, and slope in table form, they demonstrated progress towards identifying the variable relationships that correspond to such concepts. Misconceptions that arose during formative assessment were mostly

related to graph and table construction (LG1/2, Outcomes A). The only misconception that I could discover through formative assessment in relation to Outcomes B was Violet's misconception that "variable relationships are determined by the quantity of variables in an equation" (See Appendix H). This misconception seemed to be an isolated case and was, therefore, dealt with individually. My formative assessments did not provide as much information on student progress as I would have liked. Next time, I would have formulated a more focused anecdotal note taking system to collect formative assessment.

The meat of assessment analysis lies in my post assessments. Due to the personalized and detailed nature, my individual interviews created an abundance of fodder! After students constructed table and graphs of the interview exponential word problem about the number of bugs in a lake with a start value of three and growth factor of three, I asked them to look at both representations and tell me why each showed an exponential relationship. In an itemized bar graph for which each portion of the interview score is a different color, note the *green* sections that show points students earned in analysis of the exponential *table* and the *orange* sections for points earned in the analysis of the exponential *graph*.

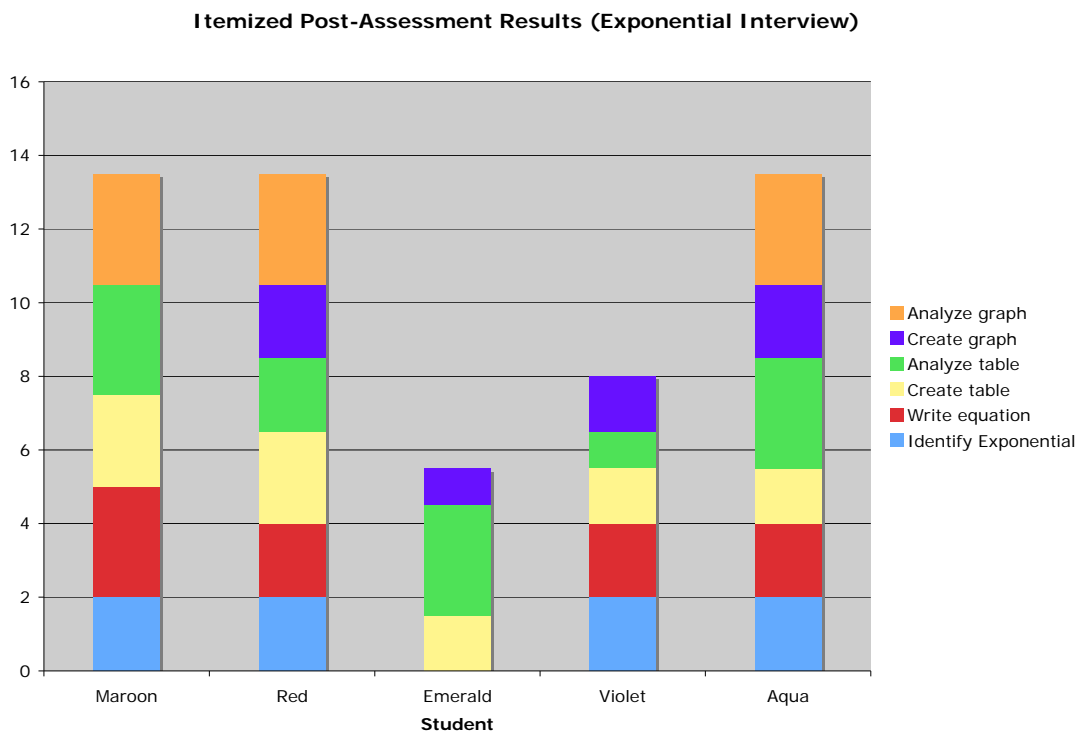**Itemized Post-Assessment Results (Exponential Interview)**

Figure 2: Post Assessment Graphic Representation 1

When noting the green sections, all five students demonstrated some amount of capability to correctly analyze exponential tables, and three out of five demonstrated some capability to correctly analyze exponential graphs. Specific rationales given for exponential tables included, "it triples," "you keep on multiplying by three," and "you multiply by the same amount." Unfortunately, only one student used the term growth factor, but every student noticed that "y values multiplied by the same value again and again" was a characteristic of exponential relationships in tables (LG1, Outcome B). Specific rationales given for exponential graphs included, "the line is curved" and "it gets larger and larger really quick and it's not a constant rate." To see 100% of my focus students achieve the table portion of LG1, Outcome B and 60% achieve the graph portion during the post-assessment interviews demonstrates a high impact upon student learning.

The second part of the personal interviews gave students the opportunity to identify and analyze a linear relationship between gallons of water in a bathtub

dependent upon the number of minutes passed when water enters at a rate of 2.5 gallons per minute. Again, I created an itemized bar graph for which each portion of the interview score is a different color. Note the *green* sections that show points students earned in analysis of the linear *table* and the *orange* sections for points earned in the analysis of the linear *graph*.

Itemized Post-Assessment Results (Linear Interview)
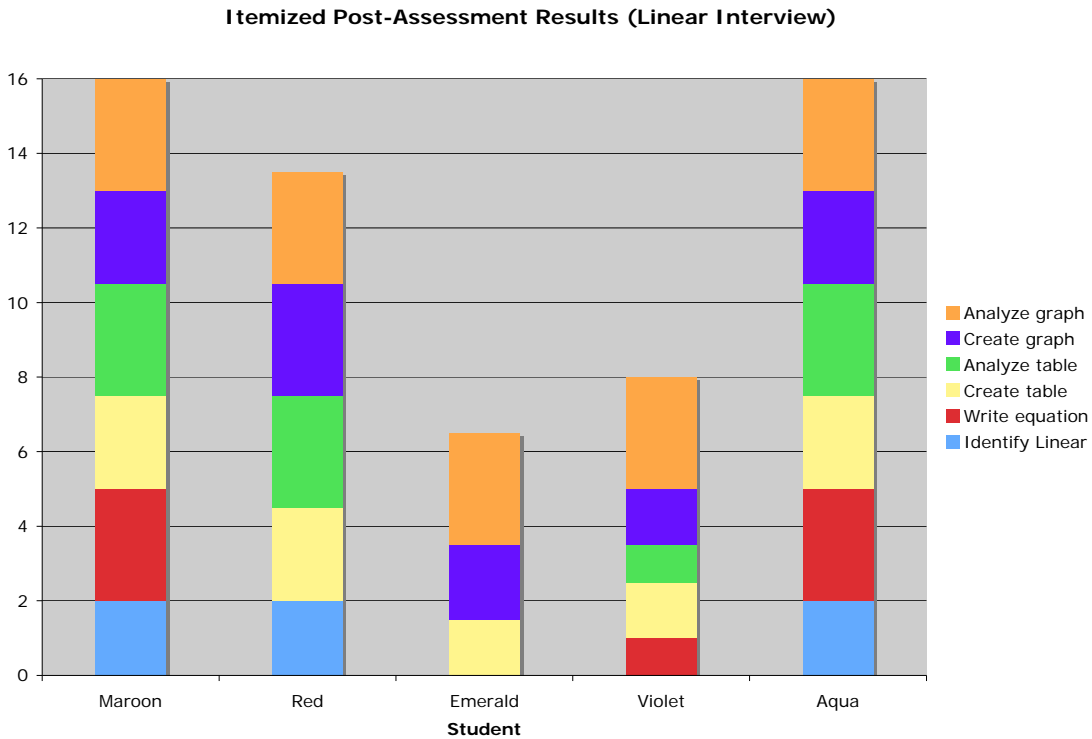
Figure 3: Post Assessment Graphic Representation 2

When noting the green sections, see that four out of five students (80%) correctly analyzed the linear table (the table part of LG2, Outcome B) and all five students correctly analyzed the linear graph (the graph part of LG2, Outcome B). *Whereas analyzing the table was a strong point for students with exponential relationships; the data suggests that students were able to identify linear relationships better in graphical form.* It is extremely fascinating that different representations were easier for students to use to identify exponential relationships versus linear relationships!

Lastly, it is valuable to analyze the items of the post assessment interview in an averaged format. Taking the average score (percentage wise) for each item in the interview, I created a line graph. Note, in particular, the discrepancy between the average "analyze table" and "analyze graph" percentages (indicated by arrows).

Average Itemized Scores
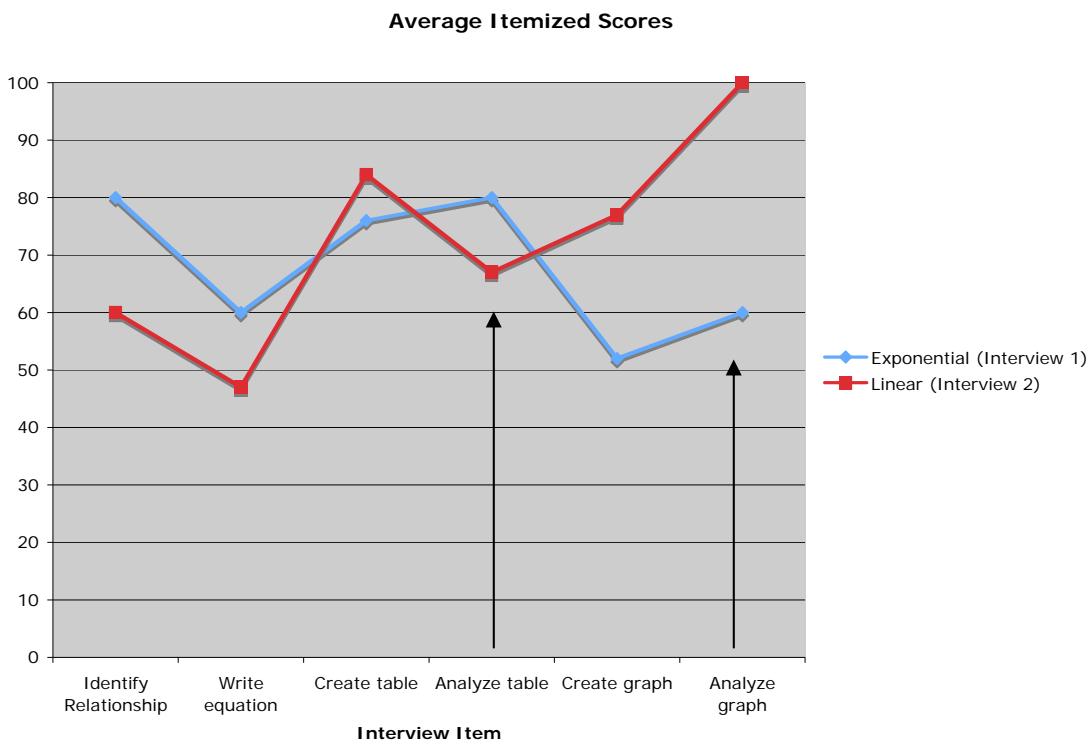
Figure 4: Post Assessment Graphic Representation 3

The above figure strongly supports the italicized statement following Figure 3. In terms of table analysis (LG1, Outcome B), the students scored 80% with the exponential situation and 67% with the linear situation. Therefore, it seems that *students are most competent with table analysis with the exponential situation*. In terms of graph analysis (LG2, Outcome B), the students scored 60% with the exponential situation and 100% with the linear situation. Therefore, it seems that the *students are most competent with the graph analysis with the linear situation*. These scores are all high in comparison with the pre-assessment data where only one out of five students (20%) had any sort of statement in regards to rationale for linear and exponential relationships.

Next Steps

Given my analysis of assessments, I have two next steps to support my focus group students' learning in regards to LG1/2, Outcomes B. First, I would provide my students with **vocabulary support** so that they could begin to use specific terms such as y-intercept, slope, start value, and growth factor in their analysis of tables and graphs. Such concepts were hinted at in the post assessment interviews, but students were not actively using the formal mathematical terms. Because the 8ᵗʰ grade mathematics EALR 4 states that, "The student communicates knowledge and understanding in both everyday and *mathematical language*," I believe that such vocabulary support would be fitting. Specifically, I would create vocabulary graphic organizers based upon the Frayer model that includes a definition, interesting fact (usually related to the etymology of the word), example, and non-example for each vocabulary word. Students would then practice and be expected to use the four terms in future analyses of tables and graphs (enriching their abilities to demonstrate LG1/2, Outcomes B).

A second next step proceeding from my assessment analysis would be to **re-teach table analysis for linear relationships**. Students had much experience identifying the slope of linear relationships in graphical and equation form in a previous unit, but the unit never emphasized the table representation. This discrepancy became apparent in my post assessment data. In order for students to reach the linear part of LG1, Outcome B, they need additional practice creating tables from linear situations, finding the slope in the table, and writing statements to support their analyses. A simple modeling of the process by myself, followed with focused practice questions would support student learning.

Individual Analysis

Introduction

Recently referred (but not yet approved) for special education services, Violet is a diligent, committed student who spends a majority of class time doing mathematics and asks many questions. She receives Title I support in mathematics, and her L1 is Spanish. I chose Violet because of the difference between the high amount of effort she puts into learning and the low amount of success that she experiences. Even more, I chose Violet because she does not show signs of noticing this difference between her effort and success. She often holds onto mathematics misconceptions and will continue applying wrong mathematics methods with confidence.

Additional Assessment Data and Analysis

Violet began the unit by filling out the warm-up activity with all sorts of misconceptions. It is admirable that she tried, however, because other students just left it blank. On the pre-assessment, Violet scored a zero. She provided a linear equation for the exponential equation, and she provided an exponential equation for the linear equation. She also provided a unique rationale for her "exponential" equation, attributing its type of relationship to the quantity of variables in the equation. Most of my formative assessment notes on her were related to LG1/2, Outcomes A, because I had to provide a lot of technical assistance in order for her to become fluent in creating tables and graphs.

To assist my analysis of Violet's progress towards LG1/2, Outcomes B, I created an itemized line graph displaying her scores on each portion of the post assessment interview. Note her scores for "analyze table" and "analyze graph" indicated by the arrows below.
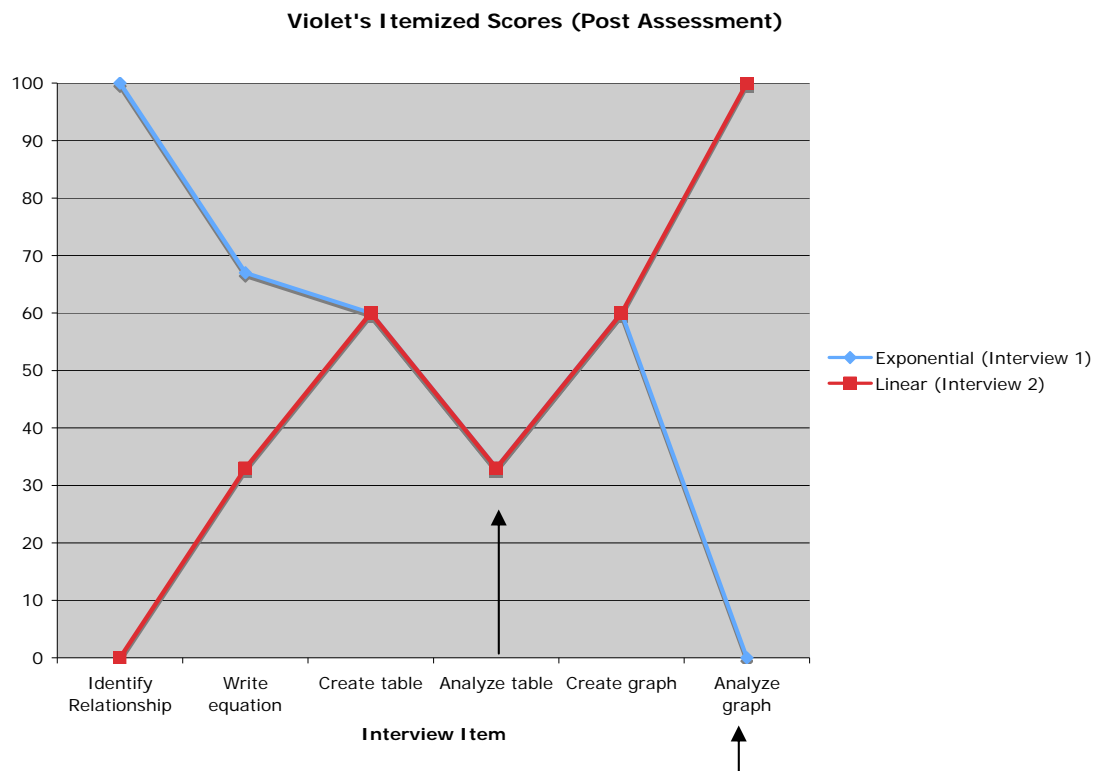
Figure 5: Post Assessment Graphic Representation (Individual)

Violet's analysis of tables was present but equally weak for both exponential and linear relationships (with a score of 33% for each). For the exponential table, she noted that, "it's going up and is exponential." She did not mention how the table was going up or hint at multiplication. Her rationale makes me wonder if she just said it was exponential because the class is working on an exponential unit. Violet's analysis of the linear table was gave me more insight into her accuracy in analyzing both linear and exponential tables. She said that the linear table was, "adding, not multiplying." (This was after I prompted her to notice what operation was occurring as the numbers "went up.") Her response was said with surprise and delight (indicating that it was more of an expression of her learning in the moment than previous learning). Violet's analysis of tables is still just forming. She noticed that increase by multiplication is characteristic of exponential tables and increase by addition is characteristic of linear tables.

Violet's analysis of graphs for variable relationships was starkly different, with a score of 0% for exponential and 100% for linear! With both graphs, Violet noted that they were "linear because they go straight up." This is true for linear graphs. The reason why Violet thought that the exponential graph was straight was because of her window settings that squished the graph so that it appeared straight. I should have corrected the window setting for her, because this construction of the graph had more to do with LG2, Outcome A and impeded my assessment of LG2, Outcome B.

More insightful than Violet's written work during the unit were the comments that she made and persistence she demonstrated during our post assessment interview (See Appendix I). It seems that Violet has developed the method of guess and check in regards to switching between equations, tables, and graphs on her graphing calculator to answer questions related to exponential and linear equations. She did not want any of my help in producing tables and graphs, and would keep entering equations until she got values that matched the handmade table we made together (for the linear relationship). Violet's persistence paid off when she eventually found the equation y=2.5x for the linear situation. Before reaching that point she threw out a bunch of exponential equations and even suggested that the relationship could be exponential decay (a new term the class had been introduced to that week).

Summary

Violet's persistence is admirable, and yet her unawareness of the reasonableness of her responses is unsettling. She did ask at the end of the interview, "What is an exponent?" That was when it became apparent to me that my assessment was not complete. Violet just "passed" the exponential interview, using exponents in equations and noting characteristics of exponential relationships in table and graph form without knowing what an exponent was. Yikes! Because Violet

has recently been referred for special education services, I do not feel as poorly about the assessment. Instead, this unawareness of reasonable answers paired with blind mathematics seems to support the referral. I am, however, very glad that Violet was able to participate in the activity and draw away conceptual ideas about variable relationships and graph/table-making skills even though she did not understand what exponents were.

Next Steps

Violet needs to learn specific strategies to assist her in meeting the 8th grade mathematics GLEs under EALR 3 (The student uses mathematical reasoning) requiring students to analyze information (3.1), conclude (3.2), and verify results (3.3). If she were taught to stop and think about the reasonableness of her responses, Violet would make less extreme mistakes. One strategy she could use would be to simply ask herself after each question, "Does this make sense?" Violet might also be taught to refer to titles or the words in a problem that clue her into what sort of mathematical concepts she is dealing with.

Violet would also benefit from a personalized mathematics dictionary where she could record her understandings of mathematical terms daily. This mathematics dictionary could then be reviewed and revised in mini-conferences with a teacher. This format would function to reveal and eliminate the many misconceptions that she creatively forms. One would need to be careful not to hinder or squelch Violet's persistence and passion to participate in mathematics. A positive comments section could be included in the mathematics dictionary to ensure that positive feedback was included.

REFLECTION AND SELF-EVALUATION
Analysis of the relationship between instruction and student learning in order to improve teaching practice

Learning Goal (Students Experienced the Most Success)

Students seemed to experience the most success with LG1, Outcome A: *Students will **use** one exponential and one linear equation to create a table of values on a graphing calculator.* Note the "Post Assessment Average Graphic Representation" (See Appendix L) where students averaged a score of 76% for creating a table with an exponential equation, and they averaged a score of 84% for creating a table with a linear equation (this being compared to average scores for creating exponential and linear graphs of 52% and 77%). Table construction skills seemed to be the most successful aspect of the unit. This might be due to the simple fact that students had been required to hand construct tables the entire year leading up to this point. To simply enter an equation and allow the graphing calculator to figure a table of values brought much delight to students. They seemed fascinated by the tables, scrolling up and down into extreme negative and extremely high x values. Student also might have experienced more success with table construction than graph construction because adjusting the table settings is fairly simple, requiring students to enter merely two values (start value and delta value).

Learning Goal (Students Experienced the Least Success)

Students seemed to experience the least success in LG2, Outcome A: *Students will **use** one exponential and one linear equation to create a graph on a graphing calculator.* Note again the "Post Assessment Average Graphic Representation" (See Appendix L) to see that students scored an average of 52% and 77% for creating exponential and linear graphs respectively. Student difficulties with the LG stemmed mostly from my lack of appropriate instruction on

how to determine reasonable window settings for a graph (a skill that I did not anticipate to be so crucial or complex). I assumed that I could merely show students how to use their tables to inform their decisions about which values to enter for the six window settings (xmin, xmax, xscl, ymin, ymax, and yscl). Only a few students grasped onto the technique of using a table to inform window settings. My instruction was not accessible to all students, leading to a lack of student success in the area of graph construction. (Note: Students had higher average scores for creating graphs with linear equations because window settings did not have to be adjusted to display a reasonable graph.) Next time I will be sure to teach this sub-skill more thoughtfully!

Possibilities for Professional Development

Professional Learning Goal 1 (PLG1): The teacher will purposefully construct questions (to be asked during both instruction and small group work time) that provoke higher order mathematical thinking in students.

I became aware of my need to be purposeful in creating questions that promote higher order mathematical thinking in students when I would ask questions that "funneled" students' responses to match my own thoughts. It seems that asking students to regurgitate my own mathematical thinking is quite opposite from encouraging them to develop their own mathematical reasoning and insights. In order to reach PLG1, I will use the questioning structures from Primary Questions and Prompts (an ATM publication) to inform my instructional design. In particular, I will use the question structures to create 1-2 higher order mathematical thinking questions per lesson and then 1-2 higher order mathematical thinking questions per group work time to ask students individually. In this way, I will build a repertoire of questions that ask students to "exemplify, specialize, complete, delete, correct, compare, sort, organize, change, vary, reverse, alter, generalize, make conjectures, explain, justify, verify, and refute" (Jeffcoat, Jones, Mansergh, and Mason, 2004).

Professional Learning Goal 2 (PLG2): The teacher will use word origins (etymology) to build mathematics vocabulary.

One of my next steps that arose from assessment analysis was to have students create graphic organizer vocabulary sheets based upon the Frayer model. Students would list the definition, characteristics, examples, and non-examples for each of the following terms: slope, y-intercept, growth factor, and start value. The "characteristics" section of the Frayer model is a great place to teach word origins. By teaching students root and affix meanings, I will enable them to connect familiar words to unfamiliar words... a powerful vocabulary tool! One professional resource that I will invest in is a mathematics word origins book, *The Words of Mathematics* by Steven Schwartzman where mathematical words are broken down into parts to determine their meanings. Etymological education for students will benefit not only their mathematical vocabularies but also enrich their general vocabularies!

Works Cited

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Lappan, G., Fey, J.T., et al. (2006). *Growing, Growing, Growing: Exponential Relationships*. Boston: Prentice Hall.

Schwartzman, S. (1994). *The Words of Mathematics: An Etymological Dictionary of Mathematical Terms Used in English*. The Mathematical Association of America.

Van de Walle, J.A. (2007). *Elementary and Middle School Mathematics*. Boston, MA: Pearson.

Appendix A: Equation Representation Rubric (ERR)/Pre-Assessment Rubric

- EE stands for exponential equation. LE stands for linear equation. Students either have the equations (1) or do not (0).
- ER stands for exponential rationale. LR stands for linear rationale. Students may have a strong (3), sufficient (2), weak (1), or non-existent (0) rationale.

Student	Warm-up 1 EE, LE, ER, LR, **Total**					Other Notes
					_/8	
					_/8	
					_/8	
					_/8	
					_/8	
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Appendix B: Formative Assessment Anecdotal Notes (FAAN)

Student	Creating a table		Identifying Exponential Relationship Characteristics in a Table	Identifying Linear Relationship Characteristics in a Table
	Strengths	Needs		

Student	Creating a graph		Identifying Exponential Relationship Characteristics in a Graph	Identifying Linear Relationship Characteristics in a Graph
	Strengths	Needs		

Appendix C: Summative Assessment Rubric for Graphing Calculator Unit

Exponential Variable Relationship Context (Problem 1)
Addresses LG2, Outcomes A and B

Assessment Component	Mastered	Still Working Towards Learning Goals	Needs Improvement	Component Point Totals
Identifies relationship as EXPONENTIAL	Yes, right away (2)	Yes, at some point during the interview (1)	No (0)	____/2
Writes a correct equation to represent the situation	Yes, perfect! (3)	Close but not quite (some components are correct) _____ (1-2)	Wrote a linear equation _____ or equation is incorrect or missing (0)	____/3
Creates a table of values from a exponential equation	Each step is worth half of a point: __ Found Y= screen __ Entered equation with y and x as variables __ Found table setting screen __ Adjusted table start and table change __ Found table screen (2.5)	Record errors that correspond to given steps:		____/2.5
Analysis of table to identify a characteristic of a exponential relationship in table representation	Characteristic is mathematically sound (generalization): Y values increase at an increasing rate. You multiply by the same amount (growth factor) over and over again. (3)	Characteristic is weak (specific to situation), but still correct: You keep on multiplying by 4. (1-2)	Characteristic is incorrect or missing. (0)	____/3
Creates a graph of values from an exponential equation	Each step is worth half of a point: __ Found Y= screen __ Entered equation with y and x as variables __ Found window screen __ Adjusted window according to table minimums and maximums for x and y __ Found graph screen (2.5)	Record errors that correspond to given steps:		___/2.5

Analysis of graph to identify a characteristic of an exponential relationship in graph representation	Characteristic is mathematically sound (generalization): I see a curved line that is increasing more and more rapidly. (3)	Characteristic is weak (specific to situation), but still correct: I see a curved line. (1-2)	Characteristic is incorrect or missing. (0)	___/3
				TOTAL: ___/16

Linear Variable Relationship Context (Problem 2)

Addresses LG1, Outcomes A and B

Assessment Component	Mastered	Still Working Towards Learning Goals	Needs Improvement	Component Point Totals
Identifies relationship as LINEAR	Yes, right away (2)	Yes, at some point during the interview (1)	No (0)	____/2
Writes a correct equation to represent the situation	Yes, perfect! (3)	Close but not quite (some components are correct) _____ (1-2)	Wrote an exponential equation _____ or equation is incorrect or missing (0)	____/3
Creates a table of values from a linear equation	Each step is worth half of a point: __ Found Y= screen __ Entered equation with y and x as variables __ Found table setting screen __ Adjusted table start and table change __ Found table screen (2.5)	Record errors that correspond to given steps:		____/2.5
Analysis of table to identify a characteristic of a linear relationship in table representation	Characteristic is mathematically sound (generalization): Y values increase at a constant/same rate, adding the same amount over and over again. (3)	Characteristic is weak (specific to situation), but still correct: You keep on adding 5. (1-2)	Characteristic is incorrect or missing. (0)	____/3
Creates a graph of values from a linear equation	Each step is worth half of a point: __ Found Y= screen __ Entered equation with y and x as variables __ Found window screen __ Adjusted window according to table minimums and maximums for x and y __ Found graph screen (2.5)	Record errors that correspond to given steps:		___/2.5

Analysis of graph to identify a characteristic of a linear relationship in graph representation	Characteristic is mathematically sound (generalization): I see a straight line increasing at a constant rate (3)	Characteristic is weak (specific to situation), but still correct: I see a straight line. (1-2)	Characteristic is incorrect or missing. (0)	___/3
				TOTAL: ___/16

GRAPHING CALCULATOR UNIT:
3 REPRESENTATIONS OF EXPONENTIAL AND LINEAR RELATIONSHIPS

Name:_____

Period:___

Day 1: Warm-up
Given the various equations on the whiteboard, complete the following statements:

1. The equation _____ represents an <u>exponential</u> relationship. I know/think this because _____ _____ _____.

2. The equation _____ represents a <u>linear</u> relationship. I know/think this because _____ _____ _____.

Characteristics of Relationships Between Variables

Representation	LINEAR RELATIONSHIPS	EXPONENTIAL RELATIONSHIPS
Equation		
Table		
Graph		

CALCULATOR TABLE REMINDER CARD

1. **Enter an equation:** Press **Y=** and enter one or more equations for Y1, Y2, Y3, etc.
    * Remember to use x as your independent variable and y as your dependent variable.
2. **Set your table settings:** Open up table settings by pressing **2nd**, **Window (TblSet)**. Choose a **TblStart** (which x-value to begin with… usually zero). Choose a "step" (or value by which x will increase) by entering a number for **delta Tbl** (usually 1,5, or 10).
3. Press **2nd**, **Graph (Table)** and the table will appear! The independent variable values are in the x column; the dependent variable values are in the Y1, Y2, Y3 columns.

CALCULATOR GRAPH REMINDER CARD

1. **Enter an equation:** Press Y= and enter one or more equations for Y1, Y2, Y3, etc.
2. **Set your graph settings:** Open up graph settings by pressing **Window**. Choose x and y minimums and maximums (*ranges*), as well as their *scales* (the mark on the x- and y-axis for every 1 unit).
3. Press **Graph** to see what the graph looks like!
4. Special feature: To find coordinates of a point on the graph, press **Trace**. Arrow along the graph and see the *coordinates* at the bottom of the screen and the *equation* in the upper left part of the screen.

Name: _____
Period: ____

Table Representations of
Exponential and Linear Relationships

Group task:
1. Choose an equation that represents an exponential relationship and one that represents a linear relationship. Record them below.
2. Enter the equation for the exponential relationship as Y1 and the equation for the linear relationship as Y2.
3. Construct a table of values (see table reminder card).

Equation that represents an <u>exponential</u> relationship: _____
Equation that represents a <u>linear</u> relationship: _____

Answer the following questions:
1. What do the values in the x column of the table represent?

2. What do the values in the Y1 column of the table represent?

3. What do the values in the Y2 column of the table represent?

4. What characteristic/quality of the Y1 column values show us that it is an <u>exponential</u> relationship?

5. What characteristic/quality of the Y2 column values show us that it is a <u>linear</u> relationship?

Appendix E: Problem 2.1 from CMP2 Student Book *Growing, Growing, Growing* (page 21)

Problem 2.1 y-Intercepts Other Than 1

Ghost Lake is a popular site for fishermen, campers, and boaters. In recent years, a certain water plant has been growing on the lake at an alarming rate. The surface area of Ghost Lake is 25,000,000 square feet. At present, 1,000 square feet are covered by the plant. The Department of Natural Resources estimates that the area is doubling every month.

A. 1. Write an equation that represents the growth pattern of the plant on Ghost Lake.

    2. Explain what information the variables and numbers in your equation represent.

    3. Compare this equation with the equations in Investigation 1.

B. 1. Make a graph of the equation.

    2. How does this graph compare with the graphs of the exponential relationships in Investigation 1?

C. How much of the lake's surface will be covered with the water plant by the end of a year?

D. In how many months will the plant completely cover the surface of the lake?

ACE Homework starts on page 24.

Investigation 2 Examining Growth Patterns **21**

56

Appendix F: Problem 2.2 from CMP2 Student Book *Growing, Growing, Growing* (page 22)

2.2 Growing Mold

Mold can spread rapidly. For example, the area covered by mold on a loaf of bread left out in warm weather grows exponentially.

Problem 2.2 Interpreting Exponential Equations

Students at Magnolia Middle School conducted an experiment. They set out a shallow pan containing a mixture of chicken bouillon (BOOL yahn), gelatin, and water. Each day, the students recorded the area of the mold in square millimeters.

The students wrote the exponential equation $m = 50(3^d)$ to model the growth of the mold. In this equation, m is the area of the mold in square millimeters after d days.

A. What is the area of the mold at the start of the experiment?

B. What is the growth factor?

C. What is the area of the mold after 5 days?

D. On which day will the area of the mold reach 6,400 mm^2?

E. An exponential equation can be written in the form $y = a(b^x)$, where a and b are constant values.

1. What value does b have in the mold equation? What does this value represent?

2. What value does a have in the mold equation? What does this value represent?

ACE Homework starts on page 24.

22   Growing, Growing, Growing

Appendix G: Post Assessment Task Sheet

Graphing Calculator Post-Assessment Task

Situation A:
As a part of a science experiment, Lucas counts 3 small bugs in a lake. After one week, he counts 9 small bugs. After two weeks, he counts 27 bugs.

1. Is the situation an example of an exponential or a linear relationship?

2. Write an equation for the number of bugs after x weeks:

3. Make a table of values on your graphing calculator with the equation.

4. Make a graph on your graphing calculator using the equation (with a realistic window).

Situation B:
A bathtub is being filled with 2.5 gallons of water per minute.

1. Is the situation an example of an exponential or a linear relationship?

2. Write an equation for the number of gallons after x minutes:

3. Make a table of values on your graphing calculator with the equation.

4. Make a graph on your graphing calculator using the equation (with a realistic window).

Appendix H: Pre, Formative, Post Assessment Compilation

Student		Pre	Post	Formative Notes
Maroon	Identify Exponential	+	+	
	Identify Linear	+	+	Provided example of linear equation
	Exponential Rationale	+	+	Identified growth factor from table rep.
	Linear Rationale	+	+	
Red	Identify Exponential	+	+	Erased correct exponential equation
	Identify Linear	0	+	
	Exponential Rationale	0	+	
	Linear Rationale	0	+	
Emerald	Identify Exponential	0	0	Left entire warm-up blank
	Identify Linear	0	0	
	Exponential Rationale	0	-	
	Linear Rationale	0	-	
Violet	Identify Exponential	0	+	Wrote linear equation for exponential equation
	Identify Linear	0	0	Wrote exponential equation for linear equation
	Exponential Rationale	0	-	Expressed misconception that variable relationships come from the number of variables in the equation
	Linear Rationale	0	-	
Aqua	Identify Exponential	0	+	Left entire warm-up blank
	Identify Linear	0	+	
	Exponential Rationale	0	+	
	Linear Rationale	0	+	

- Pre-assessment scores based upon lesson one warm-up equation activity.
- Formative assessment based upon anecdotal notes (some taken during pre- and post assessments)
- Post assessment scores based upon interviews.

Appendix I: Example of Completed Interview Rubric for Violet

"I don't do good."

RUBRIC: Exponential Variable Relationship Context
*Addresses LG2, Outcomes A and B*

Assessment Component	Mastered	Still Working Towards Learning Goals	Needs Improvement	Component Point Totals
Identifies relationship as EXPONENTIAL	Yes, right away (2) *because going by threes*	Yes, at some point during the interview (1)	No (0)	2 /2
Writes a correct equation to represent the situation	Yes, perfect! (3) *grabbed calc. when asked to write eqn.*	Close but not quite (some components are correct) $y = 3(.3x)$ (1-2) *then* $y = 3(3^x)$	Wrote a linear equation ___ or equation is incorrect or missing (0)	2 /3
Creates a table of values from a exponential equation	Each step is worth half of a point: _/ Found Y= screen ✓ Entered equation with y and x as variables ___ Found table setting screen ___ Adjusted table start and table change ✓ Found table screen (2.5)	Record errors that correspond to given steps:		1.5 /2.5
Analysis of table to identify a characteristic of a exponential relationship in table representation	Characteristic is mathematically sound (generalization): Y values increase at an increasing rate. You multiply by the same amount (growth factor) over and over again. (3)	Characteristic is weak (specific to situation), but still correct: *going* You keep on *up* multiplying by 4. (1-2) *expon. growth*	Characteristic is incorrect or missing. (0)	1 /3
Creates a graph of values from an exponential equation	Each step is worth half of a point: ✓ Found Y= screen ✓ Entered equation with y and x as variables ___ Found window screen ___ Adjusted window according to table minimums and maximums for x and y ✓ Found graph screen ___ Used trace function (3) 2.5	Record errors that correspond to given steps: *Not at first*	*with prompt → told to use window button* *Xmin=0* *Ymin=0*	1.5 /3 2.5

grabbed calc. when asked to write eqn.

Not at first

with prompt → told to use window button
Xmin=0
Ymin=0

Again, used guess and check

Analysis of graph to identify a characteristic of an exponential relationship in graph representation	Characteristic is mathematically sound (generalization): I see a curved line that is increasing more and more rapidly. (3)	Characteristic is weak (specific to situation), but still correct: I see a curved line. (1-2)	Characteristic is incorrect or missing. (0)	__0__/3
				TOTAL: 8 /16.5 16

"It's linear, going straight up"
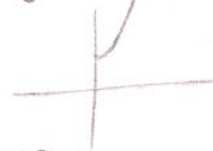
Because she saw only part of x values, so curve not very apparent

Then self-corrected: "oh, exponential b/c curved."

RUBRIC: Linear Variable Relationship Context
Addresses LG1, Outcomes A and B

| Assessment Component	Mastered	Still Working Towards Learning Goals	Needs Improvement	Component Point Totals
Identifies relationship as LINEAR	Yes, right away (2)	Yes, at some point during the interview (1)	No (0) *Expon. decay*	_0_ /2
Writes a correct equation to represent the situation	Yes, perfect! (3) *Kept trying! $y = 2.5x$ eventually*	Close but not quite (some components are correct) _____ (1-2)	Wrote an exponential equation $2.5(.5 \cap x)$ or equation is incorrect or missing (0)	_1_ /3
Creates a table of values from a linear equation	Each step is worth half of a point: ✓ Found Y= screen ✓ Entered equation with y and x as variables __ Found table setting screen __ Adjusted table start and table change ✓ Found table screen (2.5)	Record errors that correspond to given steps: *hand table- filled out $x = 2,3,4$ correctly*		1.5 /2.5
Analysis of table to identify a characteristic of a linear relationship in table representation	Characteristic is mathematically sound (generalization): Y values increase at a constant/same rate, adding the same amount over and over again. (3)	Characteristic is weak (specific to situation), but still correct: You keep on adding 5. (1-2) *w/prompt (not mult.) It's adding! → Aha! moment*	Characteristic is incorrect or missing. (0)	_1_ /3
Creates a graph of values from a linear equation	Each step is worth half of a point: ✓ Found Y= screen ✓ Entered equation with y and x as variables __ Found window screen __ Adjusted window according to table minimums and maximums for x and y ✓ Found graph screen __ Used trace function (3) 2.5	Record errors that correspond to given steps: *→ kept from last problem*		1.5 /2.5

* Very persistent. Didn't want help in finding eqn. cont. to guess and check until eqn. produced a table whose values corresponded to the hand made table we made together.

* What's an expon.?
* Used calc. expon. fct. on even when repeatedly told that not expon.

| Analysis of graph to identify a characteristic of a linear relationship in graph representation | Characteristic is mathematically sound (generalization): I see a straight line increasing at a constant rate (3) | Characteristic is weak (specific to situation), but still correct: I see a straight line. (1-2) | Characteristic is incorrect or missing. (0) | 3 /3 |
| | | | TOTAL: _8_ /16.5 |

Appendix J: Post Assessment Graphic Representation 1

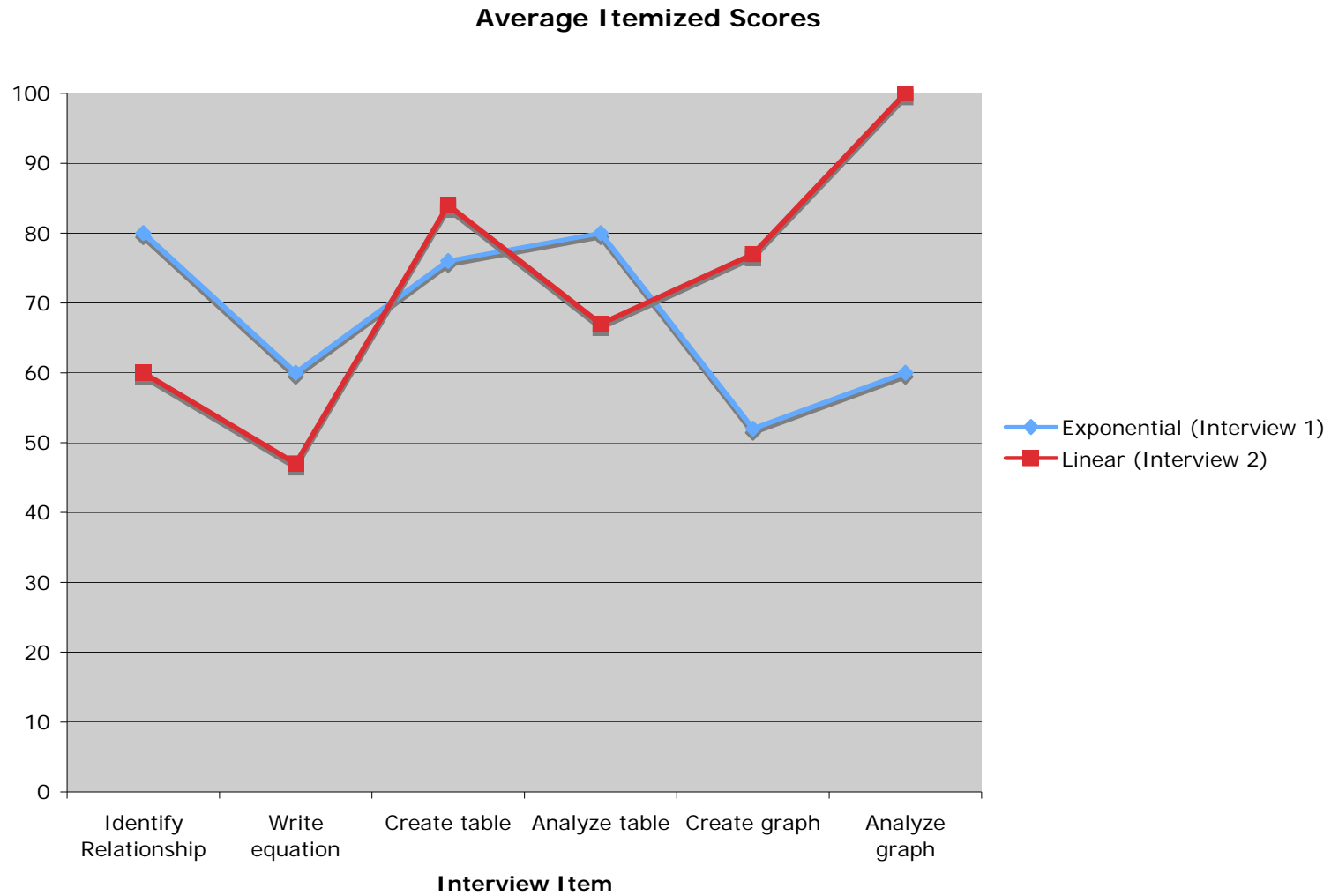**Itemized Post-Assessment Results (Exponential Interview)**
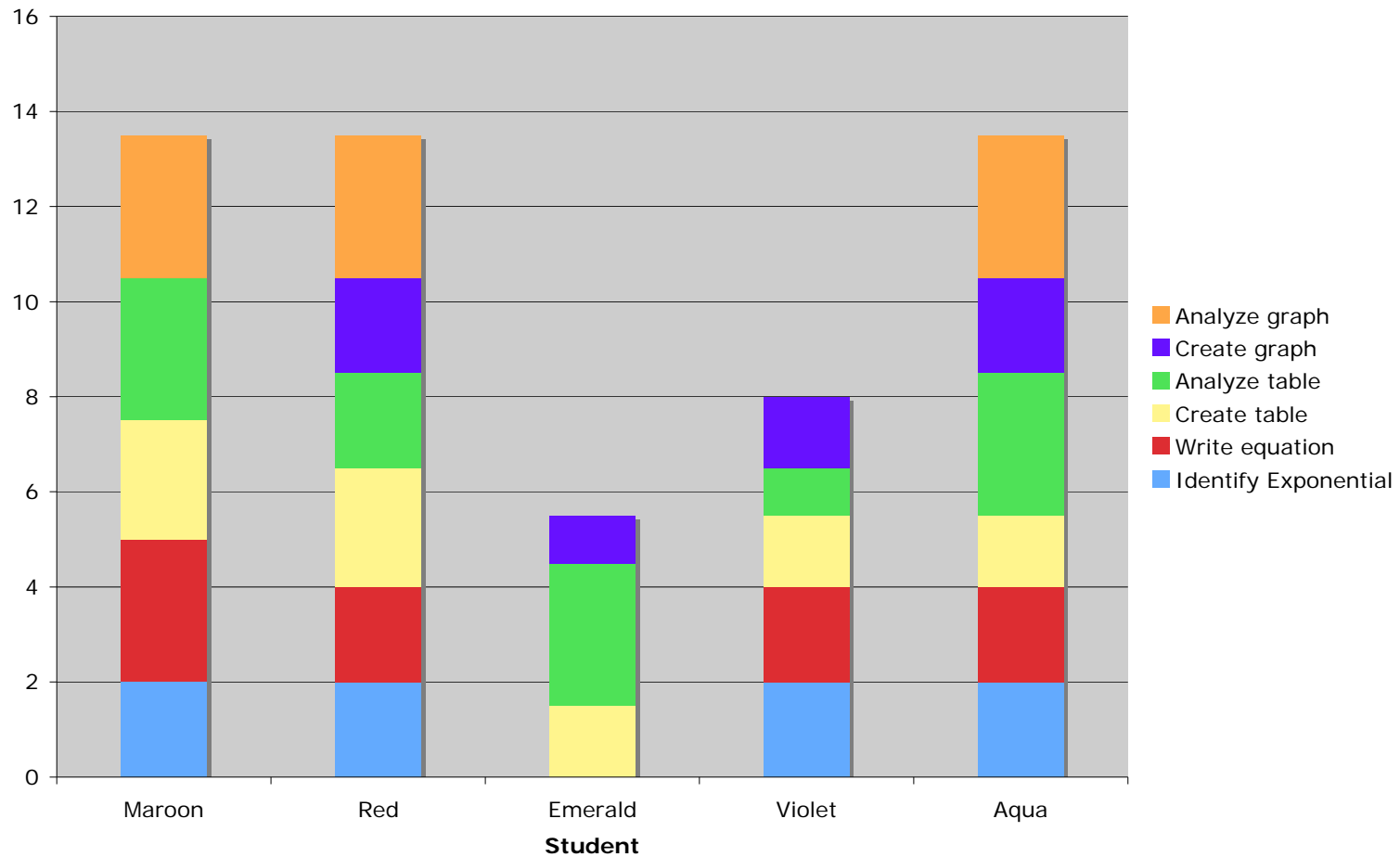
Appendix K: Post Assessment Graphic Representation 2

Itemized Post-Assessment Results (Linear Interview)
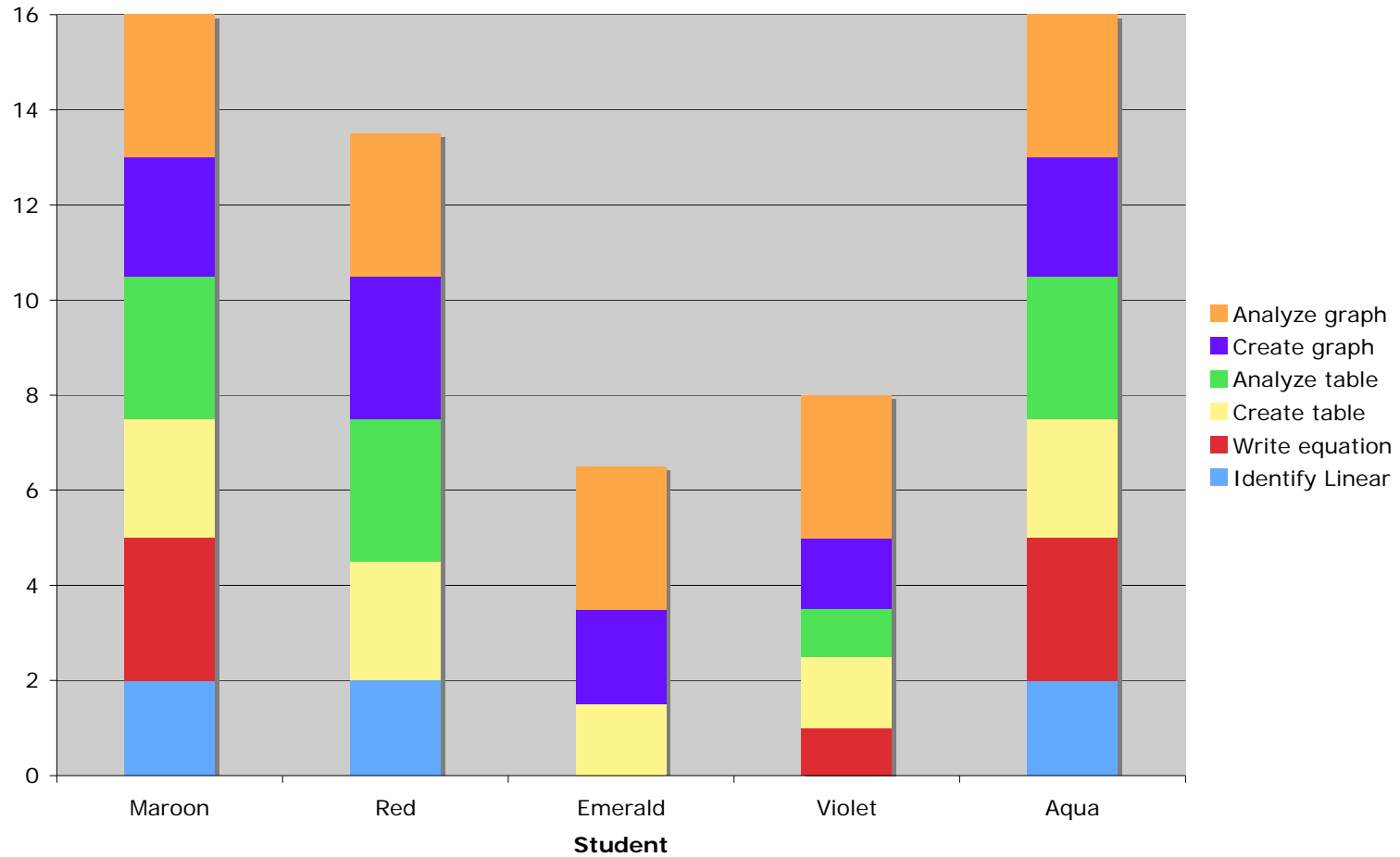
Appendix L: Post Assessment Average Graphic Representation

Average Itemized Scores

Appendix M: Post Assessment Individual Graphic Representation

Violet's Itemized Scores (Post Assessment)